

Properties of Real Numbers Guide Notes

PROPERTIES OF REAL NUMBERS

Let a , b , and c be any real numbers

1. IDENTITY PROPERTIES

A. Additive Identity

The sum of any number and 0 is equal to the number. Thus, 0 is called the **additive identity**.

For any number a , the sum of a and 0 is a .

$$a + 0 = 0 + a = a$$

B. Multiplicative Identity

The product of any number and 1 is equal to the number. Thus, 1 is called the **multiplicative identity**.

For any number a , the product of a and 1 is a .

$$a \cdot 1 = 1 \cdot a = a$$

2. INVERSE PROPERTIES

A. Additive Inverse

The sum of any number and its opposite number (its negation) is equal to 0 . Thus, 0 is called the **additive inverse**.

For any number a , the sum of a and $-a$ is 0 .

$$a + (-a) = (-a) + a = 0$$

B. Multiplicative Property of Zero

For any number a , the product of a and 0 is 0 .

$$a \cdot 0 = 0 \cdot a = 0$$

C. Multiplicative Inverse

The product of any number and its reciprocal is equal to 1 . Thus, the number's reciprocal is called the **multiplicative inverse**.

For any number a , the product of a and its reciprocal $\frac{1}{a}$ is 1 .

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

For any numbers $\frac{a}{b}$ where $b \neq 0$, the product of $\frac{a}{b}$ and its reciprocal $\frac{b}{a}$ is 1 .

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{b}{a} \cdot \frac{a}{b} = 1$$

Properties of Real Numbers Guide Notes

Sample Problem 1: Name the property in each equation. Then find the value of x .

- | | | | |
|----|---------------------------|---------------------------------------|-------------------|
| a. | $24 \cdot x = 24$ | Multiplicative identity | $x = 1$ |
| b. | $x + 0 = 51$ | Additive identity | $x = 51$ |
| c. | $x \cdot 6 = 1$ | Multiplicative inverse | $x = \frac{1}{6}$ |
| d. | $x + 19 = 0$ | Additive inverse | $x = -19$ |
| e. | $x \cdot 7 = 0$ | Multiplicative product of zero | $x = 0$ |
| f. | $\frac{3}{5} \cdot x = 1$ | Multiplicative inverse | $x = \frac{5}{3}$ |

3. EQUALITY PROPERTIES

A. Reflexive

Any quantity is equal to itself.

For any number a , $a = a$.

$$a = a$$

B. Symmetric

If one quantity equals a second quantity, then the second quantity equals the first quantity.

For any numbers a and b , if $a = b$ then $b = a$.

$$a = b$$

$$b = a$$

C. Transitive

If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.

For any numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.

$$a = b$$

$$b = c$$

$$a = c$$

D. Substitution

A quantity may be substituted for its equal in any expression.

If $a = b$, then a may be replaced by b in any expression.

$$a = b$$

$$3a = 3 \cdot b$$

Properties of Real Numbers Guide Notes

Sample Problem 2: Evaluate $x(xy - 5) + y \cdot \frac{1}{y}$, if $x = 2$ and $y = 3$. Name the property of equality used in each step.

$$\begin{aligned}
 x(xy - 5) + y \cdot \frac{1}{y} &= 2(2 \cdot 3 - 5) + 3 \cdot \frac{1}{3} && \text{Substitution: } x = 2 \text{ and } y = 3 \\
 &= 2(2 \cdot 3 - 5) + 1 && \text{Multiplicative inverse: } 3 \cdot \frac{1}{3} = 1 \\
 &= 2(6 - 5) + 1 && \text{Substitution: } 2 \cdot 3 = 6 \\
 &= 2(1) + 1 && \text{Substitution: } 6 - 5 = 1 \\
 &= 2 + 1 && \text{Multiplicative identity: } 2(1) = 2 \\
 x(xy - 5) + y \cdot \frac{1}{y} &= 3 && \text{Substitution: } 2 + 1 = 3
 \end{aligned}$$

4. COMMUTATIVE PROPERTIES

A. Addition

The order in which two numbers are added does not change their sum.

For any numbers a and b , $a + b$ is equal to $b + a$.

$$a + b = b + a$$

B. Multiplication

The order in which two numbers are multiplied does not change their product.

For any numbers a and b , $a \cdot b$ is equal to $b \cdot a$.

$$ab = ba$$

5. ASSOCIATIVE PROPERTIES

A. Addition

The way three or more numbers are grouped when adding does not change their sum.

For any numbers a , b , and c , $(a + b) + c$ is equal to $a + (b + c)$.

$$(a + b) + c = a + (b + c)$$

B. Multiplication

The way three or more numbers are grouped when multiplying does not change their product.

For any numbers a , b , and c , $(a \cdot b) \cdot c$ is equal to $a \cdot (b \cdot c)$.

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Properties of Real Numbers Guide Notes

Sample Problem 3: Simplify variable expressions. Show all possible answers.

a. $6 + (x + 3)$ $= 9 + x$ $= x + 9$

b. $(1 + x) + 2$ $= 3 + x$ $= x + 3$

c. $5 \cdot 7x$ $= 35x$

d. $(x + 4) + 8$ $= x + 12$ $= 12 + x$

e. $(6)(3x)$ $= 18x$