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Properties of Real Numbers

Unit 1 Lesson 4

Students will be able to:

Recognize and use the properties of real numbers.

Key Vocabulary:

- Identity Property
- Inverse Property
- Equality Property
- Associative Property
- Commutative Property

PROPERTIES OF REAL NUMBERS

Let $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ be any real numbers

1. IDENTITY PROPERTIES

A. **Additive Identity**

The sum of any number and $\mathbf{0}$ is equal to the number. Thus, $\mathbf{0}$ is called the **additive identity**.

For any number $\boldsymbol{a}$, the sum of $\boldsymbol{a}$ and $\mathbf{0}$ is $\boldsymbol{a}$.

$$\boldsymbol{a} + \mathbf{0} = \mathbf{0} + \boldsymbol{a} = \boldsymbol{a}$$

PROPERTIES OF REAL NUMBERS

Let $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ be any real numbers

1. IDENTITY PROPERTIES

B. **Multiplicative Identity**

The product of any number and $\mathbf{1}$ is equal to the number. Thus, $\mathbf{1}$ is called the **multiplicative identity**.

For any number $\boldsymbol{a}$, the product of $\boldsymbol{a}$ and $\mathbf{1}$ is $\boldsymbol{a}$.

$$\boldsymbol{a} \cdot \mathbf{1} = \mathbf{1} \cdot \boldsymbol{a} = \boldsymbol{a}$$

2. INVERSE PROPERTIES

A. Additive Inverse

The sum of any number and its opposite number (its negation) is equal to $\mathbf{0}$. Thus, $\mathbf{0}$ is called the **additive inverse**.

For any number $\boldsymbol{a}$, the sum of $\boldsymbol{a}$ and $-\boldsymbol{a}$ is $\mathbf{0}$.

$$\boldsymbol{a} + (-\boldsymbol{a}) = \mathbf{0}$$

$$(-\boldsymbol{a}) + \boldsymbol{a} = \mathbf{0}$$

2. INVERSE PROPERTIES

B. Multiplicative Property of Zero

For any number $\boldsymbol{a}$*, the product of* $\boldsymbol{a}$ *and* $\mathbf{0}$ *is* $\mathbf{0}$*.*

$$\boldsymbol{a} \cdot \mathbf{0} = \mathbf{0}$$

$$\mathbf{0} \cdot \boldsymbol{a} = \mathbf{0}$$

2. INVERSE PROPERTIES

C. Multiplicative Inverse

The product of any number and its reciprocal is equal to $\mathbf{1}$. Thus, the number's reciprocal is called the **multiplicative inverse**.

For any number $\boldsymbol{a}$, the product of $\boldsymbol{a}$ and its reciprocal $\frac{\mathbf{1}}{\boldsymbol{a}}$ is $\mathbf{1}$.

$$\boldsymbol{a} \cdot \frac{\mathbf{1}}{\boldsymbol{a}} = \frac{\mathbf{1}}{\boldsymbol{a}} \cdot \boldsymbol{a} = \mathbf{1}$$

For any numbers $\frac{\boldsymbol{a}}{\boldsymbol{b}}$, where $\boldsymbol{b} \neq \mathbf{0}$, the product of $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ and its reciprocal $\frac{\boldsymbol{b}}{\boldsymbol{a}}$ is $\mathbf{1}$.

$$\frac{\boldsymbol{a}}{\boldsymbol{b}} \cdot \frac{\boldsymbol{b}}{\boldsymbol{a}} = \frac{\boldsymbol{b}}{\boldsymbol{a}} \cdot \frac{\boldsymbol{a}}{\boldsymbol{b}} = \mathbf{1}$$

Sample Problem 1: Name the property in each equation. Then find the value of x.

a. $24 \cdot x = 24$

b. $x + 0 = 51$

c. $x \cdot 6 = 1$

d. $x + 19 = 0$

e. $x \cdot 7 = 0$

f. $\frac{3}{5} \cdot x = 1$

Sample Problem 1: Name the property in each equation. Then find the value of $\boldsymbol{x}$.

a.	$24 \cdot x = 24$	**Multiplicative identity**	$x = 1$
b.	$x + 0 = 51$	**Additive identity**	$x = 51$
c.	$x \cdot 6 = 1$	**Multiplicative inverse**	$x = \frac{1}{6}$
d.	$x + 19 = 0$	**Additive inverse**	$x = -19$
e.	$x \cdot 7 = 0$	**Multiplicative product of zero**	$x = 0$
f.	$\frac{3}{5} \cdot x = 1$	**Multiplicative inverse**	$x = \frac{5}{3}$

3. EQUALITY PROPERTIES

A. **Reflexive**

Any quantity is equal to itself.

For any number $\boldsymbol{a}$, $\boldsymbol{a} = \boldsymbol{a}$.

$\boldsymbol{a} = \boldsymbol{a}$

B. **Symmetric**

If one quantity equals a second quantity, then the second quantity equals the first quantity.

For any numbers $\boldsymbol{a}$ *and* $\boldsymbol{b}$, *if* $\boldsymbol{a} = \boldsymbol{b}$ *then* $\boldsymbol{b} = \boldsymbol{a}$.

$\boldsymbol{a} = \boldsymbol{b} \qquad \boldsymbol{b} = \boldsymbol{a}$

3. EQUALITY PROPERTIES

C. **Transitive**

If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.

For any numbers $\boldsymbol{a}$, $\boldsymbol{b}$, *and* $\boldsymbol{c}$, *if* $\boldsymbol{a} = \boldsymbol{b}$ *and* $\boldsymbol{b} = \boldsymbol{c}$, *then* $\boldsymbol{a} = \boldsymbol{c}$.

$\boldsymbol{a} = \boldsymbol{b}$ $\quad$ $\boldsymbol{b} = \boldsymbol{c}$

$\boldsymbol{a} = \boldsymbol{c}$

3. EQUALITY PROPERTIES

D. Substitution

A quantity may be substituted for its equal in any expression.

If $\boldsymbol{a} = \boldsymbol{b}$, then $\boldsymbol{a}$ may be replaced by $\boldsymbol{b}$ in any expression.

$$\boldsymbol{a} = \boldsymbol{b}$$

$$\boldsymbol{3a} = \boldsymbol{3} \cdot \boldsymbol{b}$$

Sample Problem 2: Evaluate $\boldsymbol{x(xy-5)+y\cdot\frac{1}{y}}$, if $\boldsymbol{x=2}$ and $\boldsymbol{y=3}$. Name the property of equality used in each step.

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Sample Problem 2: Evaluate $x(xy-5)+y\cdot\frac{1}{y}$, if $x=2$ and $y=3$.

Name the property of equality used in each step.

$x(xy-5)+y\cdot\frac{1}{y}$	$=$	$2(2\cdot 3-5)+3\cdot\frac{1}{3}$	Substitution: $x=2$ and $y=3$
	$=$	$2(2\cdot 3-5)+1$	Multiplicative inverse: $3\cdot\frac{1}{3}=1$
	$=$	$2(6-5)+1$	Substitution: $2\cdot 3=6$
	$=$	$2(1)+1$	Substitution: $6-5=1$
	$=$	$2+1$	Multiplicative identity: $2(1)=2$
$x(xy-5)+y\cdot\frac{1}{y}$	$=$	3	Substitution: $2+1=3$

4. COMMUTATIVE PROPERTIES

A. Addition

The order in which two numbers are added does not change their sum.

For any numbers $\boldsymbol{a}$ and $\boldsymbol{b}$, $\boldsymbol{a+b}$ is equal to $\boldsymbol{b+a}$.

$$\boldsymbol{a+b=b+a}$$

4. COMMUTATIVE PROPERTIES

B. **Multiplication**

The order in which two numbers are multiplied does not change their product.

For any numbers $\boldsymbol{a}$ and $\boldsymbol{b}$, $\boldsymbol{a} \cdot \boldsymbol{b}$ is equal to $\boldsymbol{b} \cdot \boldsymbol{a}$.

$$\boldsymbol{ab = ba}$$

5. ASSOCIATIVE PROPERTIES

A. Addition

The way three or more numbers are grouped when adding does not change their sum.

For any numbers $\boldsymbol{a}$, $\boldsymbol{b}$, *and* $\boldsymbol{c}$, $(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c}$ *is equal to* $\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$.

$$(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c} = \boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$$

5. ASSOCIATIVE PROPERTIES

B. Multiplication

The way three or more numbers are grouped when multiplying does not change their product.

For any numbers $\boldsymbol{a}$, $\boldsymbol{b}$, *and* $\boldsymbol{c}$, $(\boldsymbol{a} \cdot \boldsymbol{b}) \cdot \boldsymbol{c}$ *is equal to* $\boldsymbol{a} \cdot (\boldsymbol{b} \cdot \boldsymbol{c})$.

$$(\boldsymbol{a} \cdot \boldsymbol{b}) \cdot \boldsymbol{c} = \boldsymbol{a} \cdot (\boldsymbol{b} \cdot \boldsymbol{c})$$

Sample Problem 3: Simplify variable expressions. Show all possible answers.

a. $\mathbf{6 + (x + 3)}$

b. $\mathbf{(1 + x) + 2}$

c. $\mathbf{5 \cdot 7x}$

d. $\mathbf{(x + 4) + 8}$

e. $\mathbf{(6)(3x)}$

Sample Problem 3: Simplify variable expressions. Show all possible answers.

a. $6+(x+3)$ $=9+x$ $=x+9$

b. $(1+x)+2$ $=3+x$ $=x+3$

c. $5\cdot 7x$ $=35x$

d. $(x+4)+8$ $=x+12$ $=12+x$

e. $(6)(3x)$ $=18x$