



Algebra1Coach.com

Absolute Value Equations and Inequalities

Unit 3 Lesson 7

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Students will be able to:

solve absolute value equations and inequalities.

Key Vocabulary:

- Absolute Value Equations
- Absolute Value Inequalities

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

The three types of open sentences that can involve absolute value:

$$|x| = n$$

$$|x| < n$$

$$|x| > n$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

A. ABSOLUTE VALUE EQUATIONS: $|x| = n$

When solving equations involving absolute value, we need to consider these cases

- a. The value inside the absolute value symbols is **positive**.

$$x = n$$

- b. The value inside the absolute value symbols is **negative**.

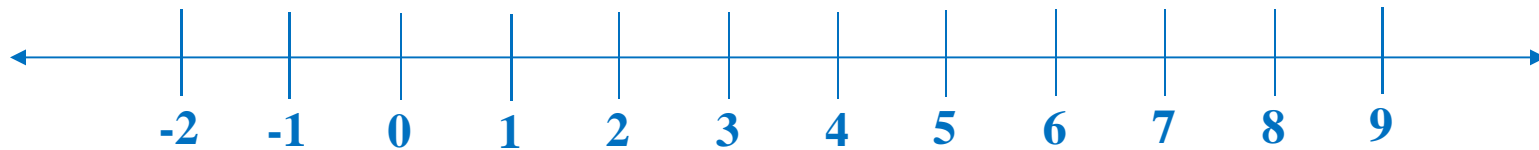
$$x = -n$$

If $|x| = n$, then $x = n$ or $x = -n$.

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

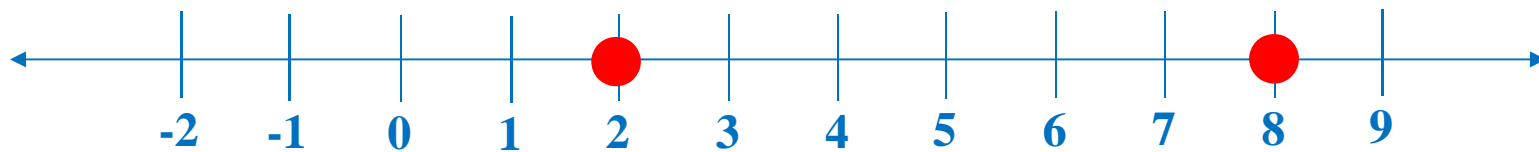
A. $|x - 5| = 3$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

A. $|x - 5| = 3$ $x = 8$ or $x = 2$



$$x - 5 = 3$$

$$x - 5 + 5 = 3 + 5$$

$$x = 8$$

$$x - 5 = -3$$

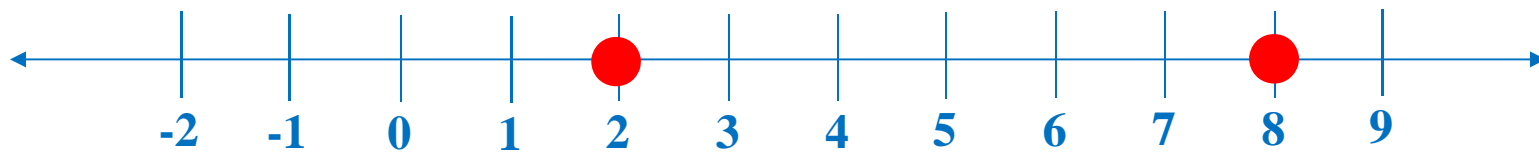
$$x - 5 + 5 = -3 + 5$$

$$x = 2$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

A. $|x - 5| = 3$ $x = 8$ or $x = 2$



Check:

$$x = 8$$

$$|8 - 5| = 3$$

$$|3| = 3$$

$$3 = 3$$

$$x = 6$$

$$|6 - 5| = 3$$

$$|1| = 3$$

$$1 \neq 3$$

$$x = 2$$

$$|2 - 5| = 3$$

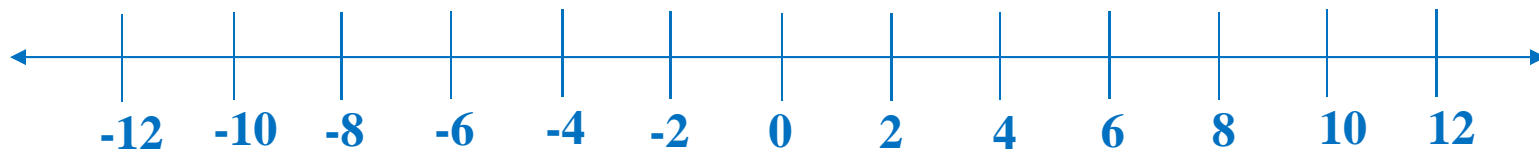
$$|-3| = 3$$

$$3 = 3$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

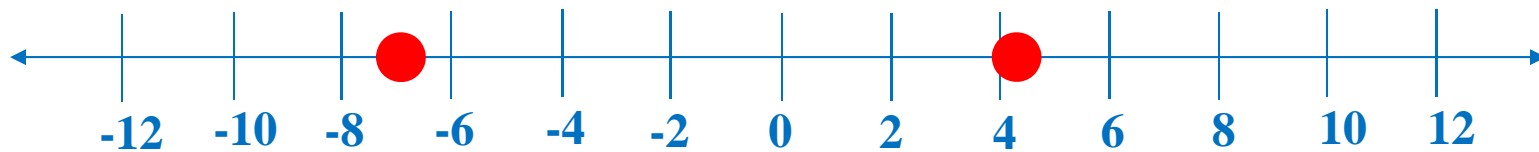
B. $|3x + 4| = 17$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

B. $|3x + 4| = 17$ $x = 13/3$ or $x = -7$



$$3x + 4 = 17$$

$$3x + 4 - 4 = 17 - 4$$

$$3x = 13$$

$$\frac{3x}{3} = \frac{13}{3}$$

$$x = 13/3$$

$$3x + 4 = -17$$

$$3x + 4 - 4 = -17 - 4$$

$$3x = -21$$

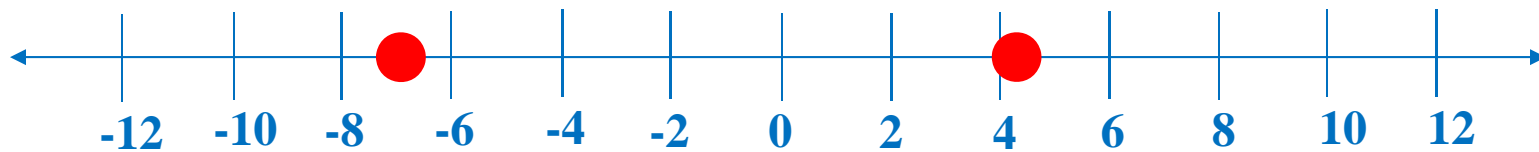
$$\frac{3x}{3} = \frac{-21}{3}$$

$$x = -7$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

B. $|3x + 4| = 17$ $x = 13/3$ or $x = -7$



Check:

$$x = 13/3$$

$$|3(13/3) + 4| = 17$$

$$|13 + 4| = 17$$

$$|17| = 17$$

$$17 = 17$$

$$x = -2$$

$$|3(-2) + 4| = 17$$

$$|-6 + 4| = 17$$

$$|-2| = 17$$

$$2 \neq 17$$

$$x = -7$$

$$|3(-7) + 4| = 17$$

$$|-21 + 4| = 17$$

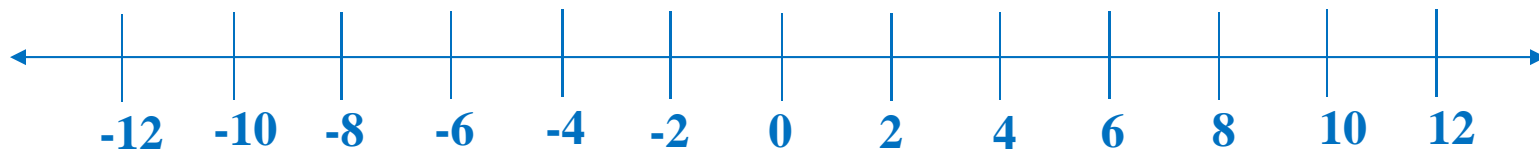
$$|-17| = 17$$

$$17 = 17$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

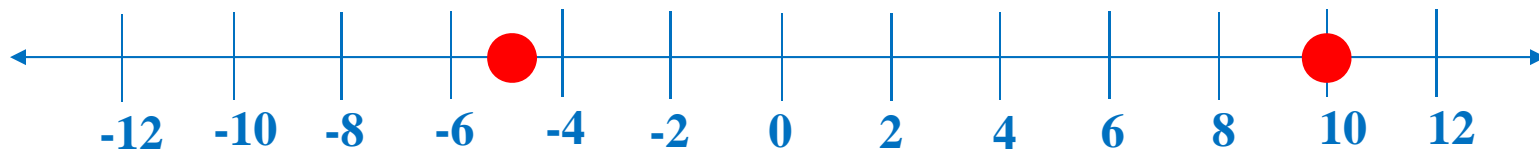
C. $|2x - 5| - 9 = 6$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

C. $|2x - 5| - 9 = 6$ $x = 10$ or $x = -5$



$$|2x - 5| - 9 + 9 = 6 + 9$$

$$|2x - 5| = 15$$

$$|2x - 5| = 15$$

$$2x - 5 = 15$$

$$2x - 5 + 5 = 15 + 5$$

$$2x = 20$$

$$\frac{2x}{2} = \frac{20}{2}$$

$$x = 10$$

$$|2x - 5| = -15$$

$$2x - 5 = -15$$

$$2x - 5 + 5 = -15 + 5$$

$$2x = -10$$

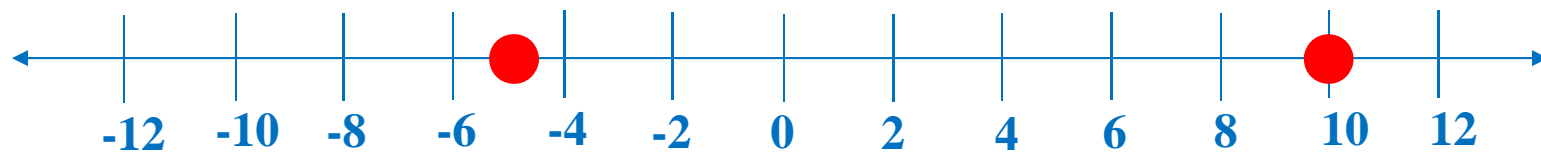
$$\frac{2x}{2} = \frac{-10}{2}$$

$$x = -5$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 1: Solve each equation and graph the solution set.

C. $|2x - 5| - 9 = 6$ $x = 10$ or $x = -5$



Check:

$$x = 10$$

$$|2(\mathbf{10}) - 5| - 9 = 6$$

$$|20 - 5| - 9 = 6$$

$$|15| - 9 = 6$$

$$15 - 9 = 6$$

$$6 = 6$$

$$x = 2$$

$$|2(\mathbf{2}) - 5| - 9 = 6$$

$$|4 - 5| - 9 = 6$$

$$|-1| - 9 = 6$$

$$1 - 9 = 6$$

$$-8 \neq 6$$

$$x = -5$$

$$|2(\mathbf{-5}) - 5| - 9 = 6$$

$$|-10 - 5| - 9 = 6$$

$$|-15| - 9 = 6$$

$$15 - 9 = 6$$

$$6 = 6$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

B. ABSOLUTE VALUE INEQUALITIES: $|x| < n$

When solving inequalities of the form $|x| < n$, we need to find the **Intersection** of these cases:

- The value inside the absolute value symbols is **less than the positive value of n** .

$$x < n$$

- The value inside the absolute value symbols is **greater than the negative value of n** .

$$x > -n$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

B. ABSOLUTE VALUE INEQUALITIES: $|x| < n$

If $|x| < n$, then $x < n$ and $x > -n$.

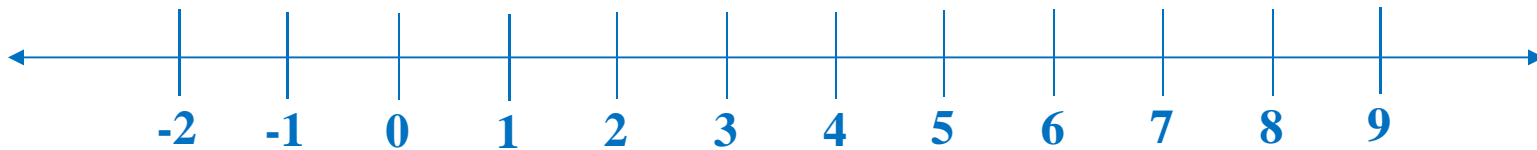
It also applies for $|x| \leq n$:

If $|x| \leq n$, then $x \leq n$ and $x \geq -n$.

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

A. $|x - 6| < 2$

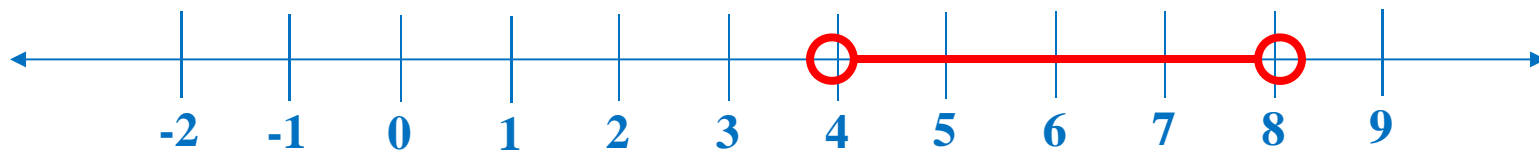


ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

A. $|x - 6| < 2$

$$4 < x < 8$$



$$x - 6 < 2$$

$$x - 6 > -2$$

$$x - 6 + 6 < 2 + 6$$

$$x - 6 + 6 > -2 + 6$$

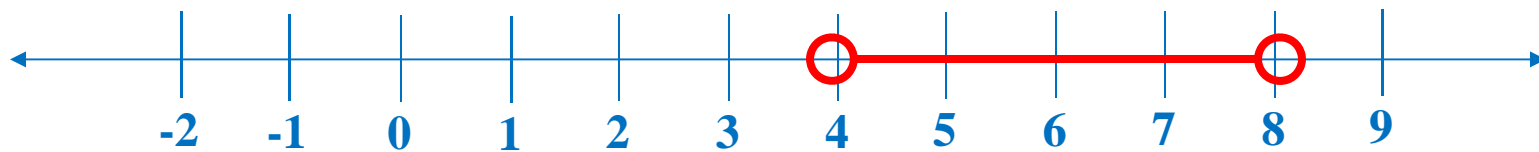
$$x < 8$$

$$x > 4$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

A. $|x - 6| < 2$ $4 < x < 8$



Check:

$x < 8$

$x = 7$

$|7 - 6| < 2$

$|1| < 2$

$1 < 2$

$x = 8$

$|8 - 6| < 2$

$|8| < 2$

$2 < 2$

$x = 9$

$|9 - 6| < 2$

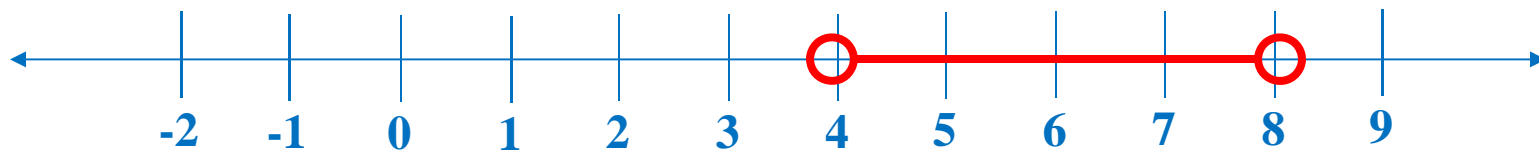
$|3| < 2$

$3 \not< 2$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

A. $|x - 6| < 2$ $4 < x < 8$



Check: $x > 4$

$$x = 3$$

$$|3 - 6| < 2$$

$$|-3| < 2$$

$$3 \not< 2$$

$$x = 4$$

$$|4 - 6| < 2$$

$$|-4| < 2$$

$$2 \not< 2$$

$$x = 5$$

$$|5 - 6| < 2$$

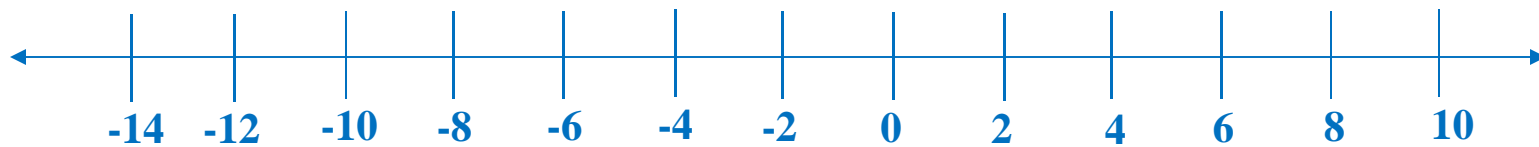
$$|-1| < 2$$

$$1 < 2$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

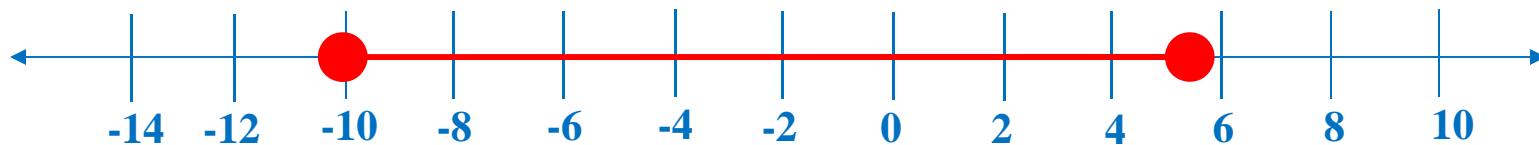
B. $|3x + 7| \leq 23$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

B. $|3x + 7| \leq 23$ $-10 \leq x \leq 16/3$



$$3x + 7 \leq 23$$

$$3x + 7 \geq -23$$

$$3x + 7 - 7 \leq 23 - 7 \quad 3x + 7 - 7 \geq -23 - 7$$

$$3x \leq 16$$

$$3x \geq -30$$

$$\frac{3x}{3} \leq \frac{16}{3}$$

$$\frac{3x}{3} \geq \frac{-30}{3}$$

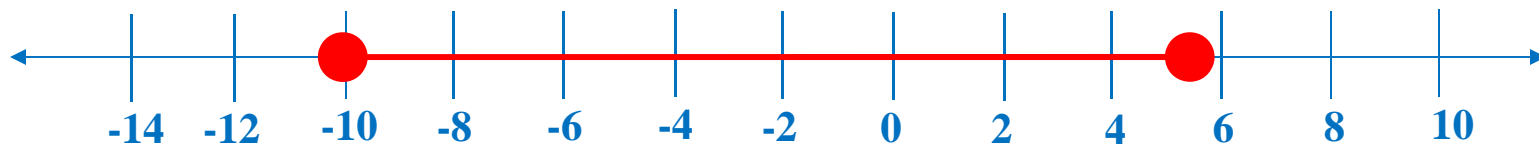
$$x \leq 16/3$$

$$x \geq -10$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

B. $|3x + 7| \leq 23$ $-10 \leq x \leq 16/3$



Check: $x \leq 16/3$

$$x = 5$$

$$|3(5) + 7| \leq 23$$

$$|15 + 7| \leq 23$$

$$|22| \leq 23$$

$$22 \leq 23$$

$$x = 16/3$$

$$|3(16/3) + 7| \leq 23$$

$$|16 + 7| \leq 23$$

$$|23| \leq 23$$

$$23 \leq 23$$

$$x = 6$$

$$|3(6) + 7| \leq 23$$

$$|18 + 7| \leq 23$$

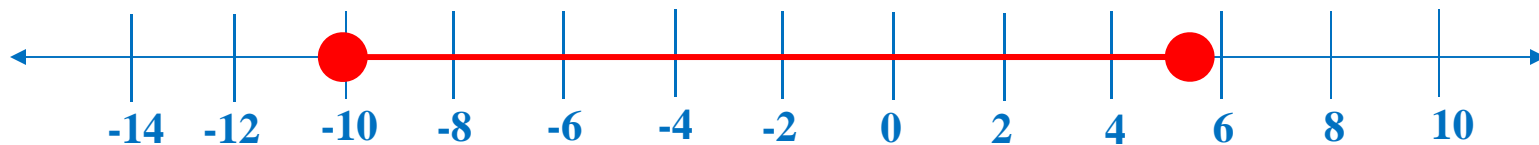
$$|25| \leq 23$$

$$25 \not\leq 23$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

B. $|3x + 7| \leq 23$ $-10 \leq x \leq 16/3$



Check: $x \geq -10$

$$x = -11$$

$$|3(-11) + 7| \leq 23$$

$$|-33 + 7| \leq 23$$

$$|-26| \leq 23$$

$$26 \not\leq 23$$

$$x = -10$$

$$|3(-10) + 7| \leq 23$$

$$|-30 + 7| \leq 23$$

$$|-23| \leq 23$$

$$23 \leq 23$$

$$x = -9$$

$$|3(-9) + 7| \leq 23$$

$$|-27 + 7| \leq 23$$

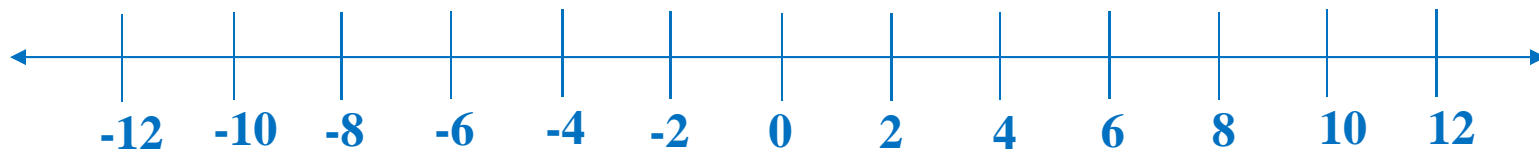
$$|-20| \leq 23$$

$$20 \leq 23$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

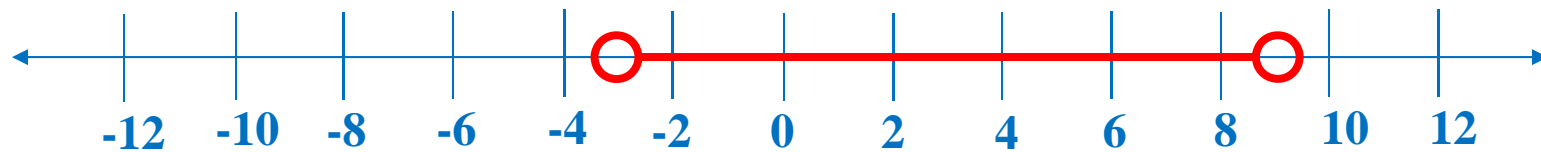
C. $|3x - 9| - 4 < 14$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

C. $|3x - 9| - 4 < 14$ $-3 < x < 9$



$$|3x - 9| - 4 + 4 < 14 + 4$$

$$|3x - 9| < 18$$

$$3x - 9 < 18$$

$$3x - 9 + 9 < 18 + 9$$

$$3x < 27$$

$$\frac{3x}{3} < \frac{27}{3}$$

$$x < 9$$

$$3x - 9 > -18$$

$$3x - 9 + 9 > -18 + 9$$

$$3x > -9$$

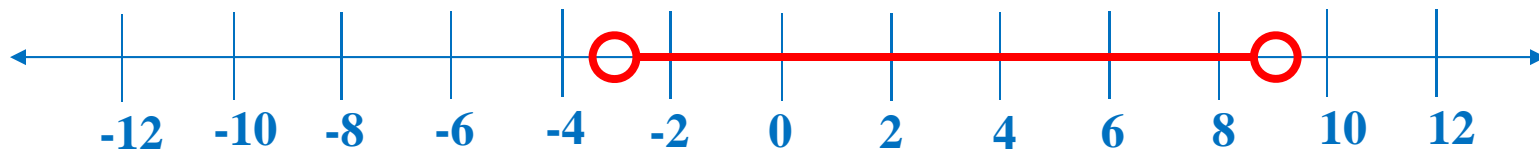
$$\frac{3x}{3} > \frac{-9}{3}$$

$$x > -3$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

C. $|3x - 9| - 4 < 14$ $-3 < x < 9$



Check: $x < 9$

$x = 8$

$$|3(8) - 9| - 4 < 14$$

$$|24 - 9| - 4 < 14$$

$$|15| - 4 < 14$$

$$15 - 4 < 14$$

$$11 < 14$$

$x = 9$

$$|3(9) - 9| - 4 < 14$$

$$|27 - 9| - 4 < 14$$

$$|18| - 4 < 14$$

$$18 - 4 < 14$$

$$14 \not< 14$$

$x = 10$

$$|3(10) - 9| - 4 < 14$$

$$|30 - 9| - 4 < 14$$

$$|21| - 4 < 14$$

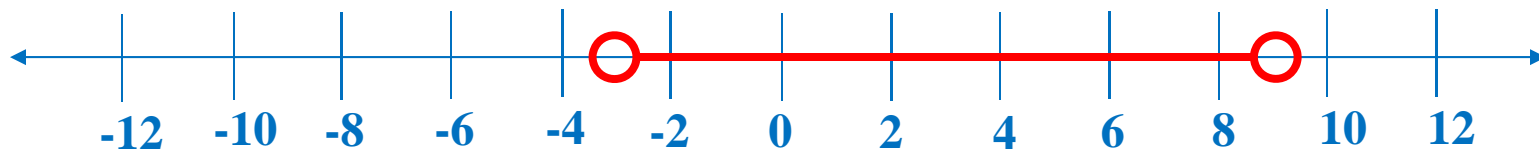
$$15 - 4 < 14$$

$$17 \not< 14$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 2: Solve each equation and graph the solution set.

C. $|3x - 9| - 4 < 14$ $-3 < x < 9$



Check: $x > -3$

$$x = -4$$

$$|3(-4) - 9| - 4 < 14$$

$$|-12 - 9| - 4 < 14$$

$$|-21| - 4 < 14$$

$$21 - 4 < 14$$

$$17 \not< 14$$

$$x = -3$$

$$|3(-3) - 9| - 4 < 14$$

$$|-9 - 9| - 4 < 14$$

$$|-18| - 4 < 14$$

$$18 - 4 < 14$$

$$14 \not< 14$$

$$x = -2$$

$$|3(-2) - 9| - 4 < 14$$

$$|-6 - 9| - 4 < 14$$

$$|-15| - 4 < 14$$

$$15 - 4 < 14$$

$$11 < 14$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

C. ABSOLUTE VALUE INEQUALITIES: $|x| > n$

When solving inequalities of the form $|x| > n$, we need to find the **Intersection** of these cases:

a. The value inside the absolute value symbols is **GREATER THAN THE POSITIVE VALUE OF n** .

$$x > n$$

b. The value inside the absolute value symbols is **LESS THAN THE NEGATIVE VALUE OF n** .

$$x < -n$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

C. ABSOLUTE VALUE INEQUALITIES: $|x| > n$

If $|x| > n$, then $x > n$ or $x < -n$.

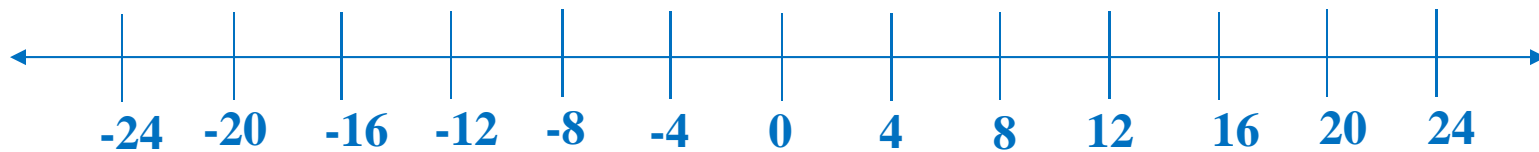
It also applies for $|x| \geq n$:

If $|x| \geq n$, then $x \geq n$ or $x \leq -n$.

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

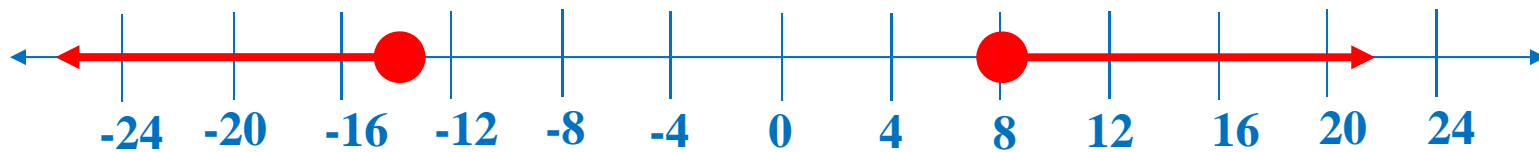
A. $|x + 3| \geq 11$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

A. $|x + 3| \geq 11$ $x \geq 8$ or $x \leq -14$



$$x + 3 \geq 11$$

$$x + 3 - 3 \geq 11 - 3$$

$$x \geq 8$$

$$x + 3 \leq -11$$

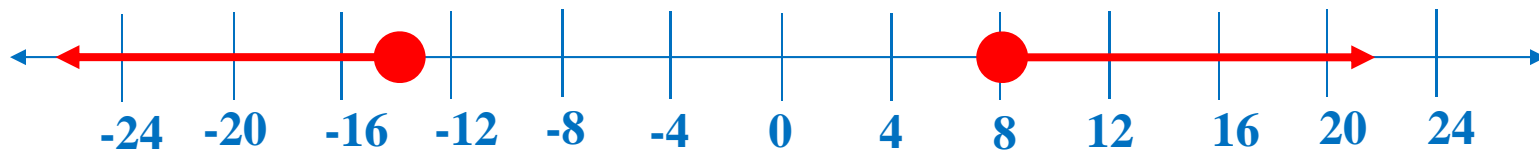
$$x + 3 - 3 \leq -11 - 3$$

$$x \leq -14$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

A. $|x + 3| \geq 11$ $x \geq 8$ or $x \leq -14$



Check:

$x \geq 8$

$x = 7$

$|7 + 3| \geq 11$

$|10| \geq 11$

$10 \not\geq 11$

$x = 8$

$|8 + 3| \geq 11$

$|11| \geq 11$

$11 \geq 11$

$x = 9$

$|9 + 3| \geq 11$

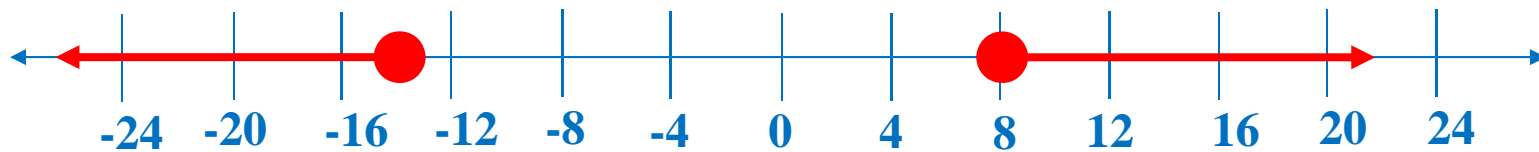
$|12| \geq 11$

$12 \geq 11$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

A. $|x + 3| \geq 11$ $x \geq 8$ or $x \leq -14$



Check:

$x \leq -14$

$x = -13$

$|-13 + 3| \geq 11$

$|-10| \geq 11$

$10 \not\geq 11$

$x = -14$

$|-14 + 3| \geq 11$

$|-11| \geq 11$

$11 \geq 11$

$x = -15$

$|-15 + 3| \geq 11$

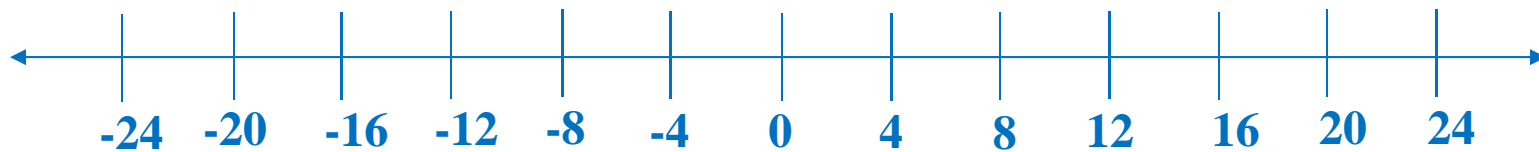
$|-12| \geq 11$

$12 \geq 11$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

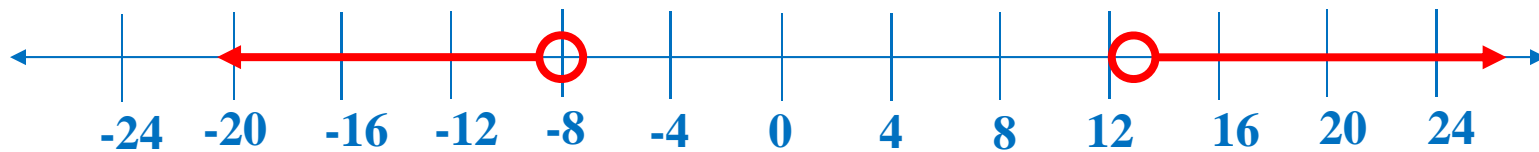
B. $|2x - 5| > 21$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

B. $|2x - 5| > 21$ $x > 13$ or $x < -8$



$$2x - 5 > 21$$

$$2x - 5 + 5 > 21 + 5$$

$$2x > 26$$

$$\frac{2x}{2} > \frac{26}{2}$$

$$x > 13$$

$$2x - 5 < -21$$

$$2x - 5 + 5 < -21 + 5$$

$$2x < -16$$

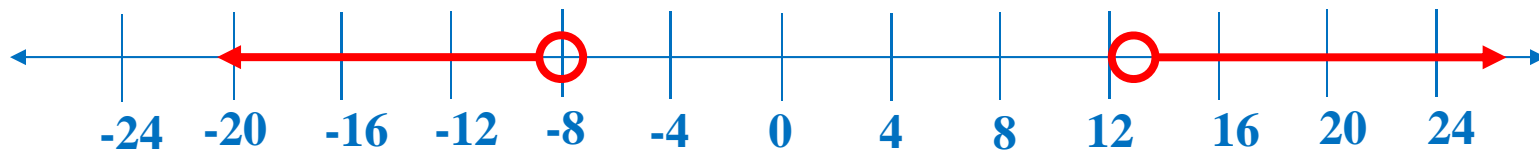
$$\frac{2x}{2} < \frac{-16}{2}$$

$$x < -8$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

B. $|2x - 5| > 21$ $x > 13$ or $x < -8$



Check: $x > 13$

$$x = 12$$

$$|2(\mathbf{12}) - 5| > 21$$

$$|24 - 5| > 21$$

$$|19| > 21$$

$$19 \not> 21$$

$$x = 13$$

$$|2(\mathbf{13}) - 5| > 21$$

$$|26 - 5| > 21$$

$$|21| > 21$$

$$21 \not> 21$$

$$x = 14$$

$$|2(\mathbf{14}) - 5| > 21$$

$$|28 - 5| > 21$$

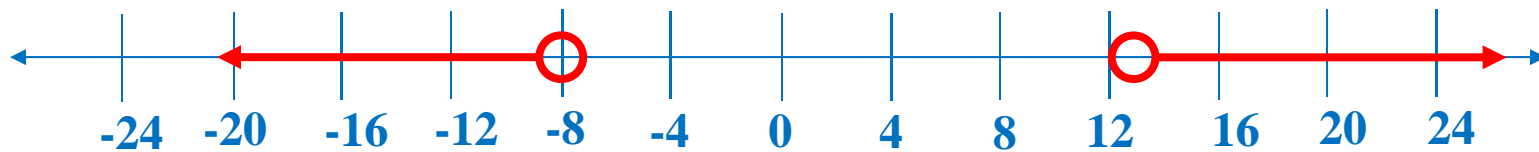
$$|23| > 21$$

$$23 > 21$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

B. $|2x - 5| > 21$ $x > 13$ or $x < -8$



Check: $x < -8$

$$x = -7$$

$$|2(-7) - 5| > 21$$

$$|-14 - 5| > 21$$

$$|-19| > 21$$

$$19 \not> 21$$

$$x = -8$$

$$|2(-8) - 5| > 21$$

$$|-16 - 5| > 21$$

$$|-21| > 21$$

$$21 \not> 21$$

$$x = -9$$

$$|2(-9) - 5| > 21$$

$$|-18 - 5| > 21$$

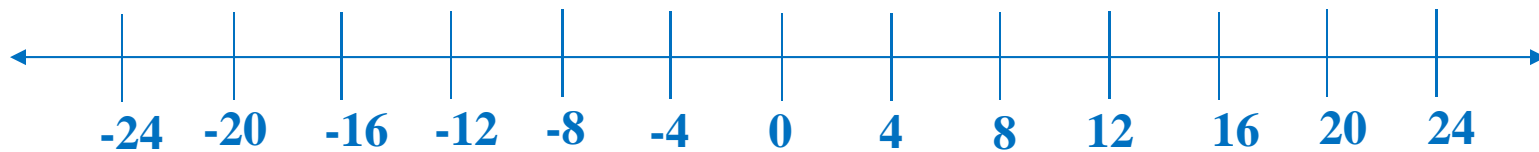
$$|-23| > 21$$

$$23 > 21$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

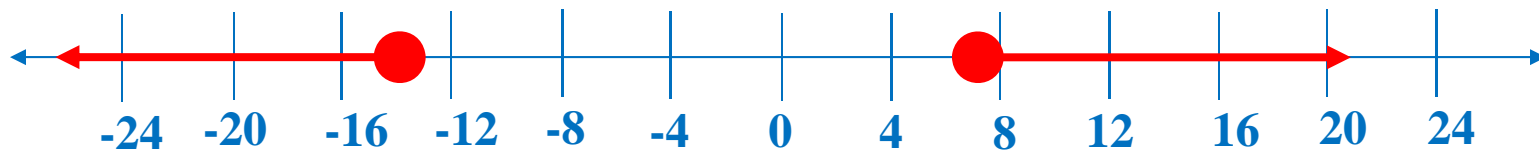
C. $|2x + 7| - 6 \geq 15$



ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

C. $|2x + 7| - 6 \geq 15$ $x \geq 7$ or $x \leq -14$



$$|2x + 7| - 6 + 6 \geq 15 + 6$$

$$|2x + 7| \geq 21$$

$$2x + 7 \geq 21$$

$$2x + 7 - 7 \geq 21 - 7$$

$$2x \geq 14$$

$$\frac{2x}{2} \geq \frac{14}{2}$$

$$x \geq 7$$

$$2x + 7 \leq -21$$

$$2x + 7 - 7 \leq -21 - 7$$

$$2x \leq -28$$

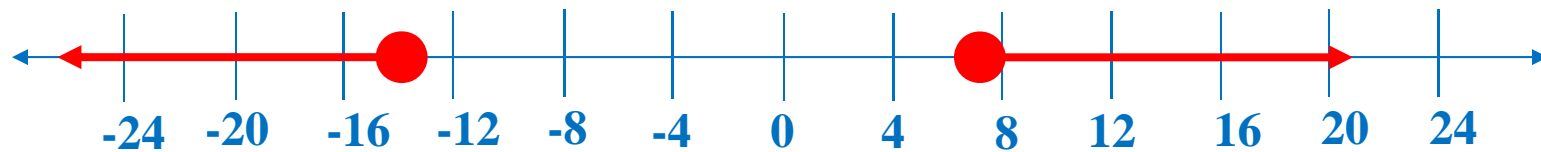
$$\frac{2x}{2} \leq \frac{-28}{2}$$

$$x \leq -14$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

C. $|2x + 7| - 6 \geq 15$ $x \geq 7$ or $x \leq -14$



Check: $x \geq 7$

$$x = 6$$

$$|2(6) + 7| - 6 \geq 15$$

$$|12 + 7| - 6 \geq 15$$

$$|19| - 6 \geq 15$$

$$19 - 6 \geq 15$$

$$13 \geq 15$$

$$x = 7$$

$$|2(7) + 7| - 6 \geq 15$$

$$|14 + 7| - 6 \geq 15$$

$$|21| - 6 \geq 15$$

$$21 - 6 \geq 15$$

$$15 \geq 15$$

$$x = 8$$

$$|2(8) + 7| - 6 \geq 15$$

$$|16 + 7| - 6 \geq 15$$

$$|23| - 6 \geq 15$$

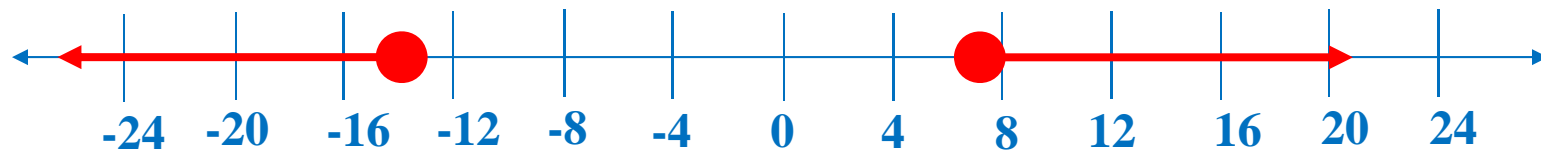
$$23 - 6 \geq 15$$

$$17 \geq 15$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 3: Solve each equation and graph the solution set.

C. $|2x + 7| - 6 \geq 15$ $x \geq 7$ or $x \leq -14$



Check: $x \leq -14$

$$x = -13$$

$$|2(-13) + 7| - 6 \geq 15$$

$$|-26 + 7| - 6 \geq 15$$

$$|-19| - 6 \geq 15$$

$$19 - 6 \geq 15$$

$$13 \geq 15$$

$$x = -14$$

$$|2(-14) + 7| - 6 \geq 15$$

$$|-28 + 7| - 6 \geq 15$$

$$|-21| - 6 \geq 15$$

$$21 - 6 \geq 15$$

$$15 \geq 15$$

$$x = -15$$

$$|2(-15) + 7| - 6 \geq 15$$

$$|-30 + 7| - 6 \geq 15$$

$$|-23| - 6 \geq 15$$

$$23 - 6 \geq 15$$

$$17 \geq 15$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 4: The starting players of the school's varsity basketball team have an average scoring point between 9 and 20. Write an absolute value inequality describing the average scoring point of the starting players.

Step 1 Write the inequality.

Step 2 Determine the midpoint.

Step 3 Determine the distance from the midpoint.

Step 4 Write the absolute value inequality. (Use appropriate inequality symbol)

$$|x - \mathit{midpoint}| < \mathit{distance\ from\ the\ midpoint}$$

ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Sample Problem 4: The starting players of the school's varsity basketball team have an average scoring point between 9 and 20. Write an absolute value inequality describing the average scoring point of the starting players.

Step 1 Write the inequality.

$$9 < x < 20$$

Step 2 Determine the midpoint.

$$\frac{9 + 20}{2} = \frac{29}{2} = 14.5$$

Step 3 Determine the distance from the midpoint.

$$|20 - 14.5| < 5.5 \quad |9 - 14.5| < 5.5$$

Step 4 Write the absolute value inequality. (Use appropriate inequality symbol)

$$|x - \textit{midpoint}| < \textit{distance from the midpoint}$$

$$|x - 14.5| < 5.5$$