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Simplifying Radicals Guided Notes

Radicals (or roots) are the opposite operation of applying exponents. A power can be undone with a radical and a radical can be undone with a power.

$2^2 = 4 \quad so \ \sqrt{4} = 2$

$2^3 = 8 \quad so \ \sqrt[3]{8} = 2$

$\sqrt[index]{radicand}$

Radical sign

Common Radicals:

A square (second) root is written as $\sqrt{\ }$

A cube (third) root is written as $\sqrt[3]{\ }$

A fourth root is written as $\sqrt[4]{\ }$

Simplifying Radicals

To simplify a radical, factor the expression under the radical sign to its prime factors. For every pair of like factors, bring out one of the factors. Multiply whatever is outside the sign, and then multiply whatever is inside the sign. Remember that for each pair, you “bring out” only one of the numbers.

Variables in a radicand are simplified in the same way. You've got a pair of can be taken "out front".

Negative Radicals – The only restriction that exists for negative signs and radicals is that there cannot be a negative sign under an even root since there is no real solution to this problem.

However, a negative sign can exist in front of a radical or under odd roots and still be able to obtain a real number.

- **PRODUCT RULE FOR RADICAL**
  The radical of the product is the product of two radicals.
  $\sqrt[m]{xy} = \sqrt[m]{x} * \sqrt[m]{y}$

- **QUOTIENT RULE FOR RADICAL**
  The radical of a quotient is the quotient of the radicals.
  $\sqrt[m]{\frac{x}{y}} = \frac{\sqrt[m]{x}}{\sqrt[m]{y}} \qquad y \neq 0$

Sample Problem 1: Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\sqrt{20} =$

b. $\sqrt{72a^2} =$

c. $\sqrt{45} =$

d. $\sqrt{\frac{24}{54}} =$

e. $\sqrt[5]{\frac{-32y^5}{x^{10}}} =$

f. $\sqrt[3]{\frac{xz^4}{y^6}} =$

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Simplifying Radicals Guided Notes

- Radicals can also be written in exponent notation. However, in this case the exponent would be a fraction.

  $\sqrt[m]{x^n} = x^{\frac{n}{m}}$

  If n is odd then $\sqrt[n]{x^n} = x$

  If n is even then $\sqrt[n]{x^n} = |x|$

Sample Problem 2:

Write each expression in radical notation.

a. $(4)^{\frac{1}{2}} =$ **b.** $(-8)^{\frac{1}{3}} =$ **c.** $4^{\frac{1}{3}} =$

Sample Problem 3:

Write each expression in exponential notation.

a. $\sqrt[3]{(-64)} =$ **b.** $\sqrt[5]{32} =$ **c.** $\sqrt[4]{625} =$

- **RATIONALIZING THE DENOMINATOR**

  1. When dealing with fractions, a final answer cannot contain radicals in the denominator; therefore, it is necessary to eliminate any radical from the denominator. The process of removing the radical from the denominator is called **rationalizing the denominator**.

  A radical is considered simplified only if there is no radical sign in the denominator.

$$\frac{1}{\sqrt[n]{x^m}} = \frac{1}{\sqrt[n]{x^m}} * \frac{\sqrt[n]{x^{n-m}}}{\sqrt[n]{x^{n-m}}} = \frac{\sqrt[n]{x^{n-m}}}{x}$$

  2. Rationalizing the denominator with two terms, one or both of which involve square root.

  Step 1: Multiply the numerator and denominator by the conjugate of the denominator

  Step 2: Simplify the resulting expression if possible.

Sample Problem 4: Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\frac{7}{\sqrt{5}} =$ **b.** $\frac{2}{\sqrt{(x+y)}} =$

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Simplifying Radicals Guided Notes

c. $\frac{2}{2-\sqrt{6}} =$

d. $\frac{x}{1-\sqrt{x}} =$

- **POWER OF A RADICAL**
  The radicand has to be raised to the exponent of the power while the index remains the same.

  $$\left(\sqrt[m]{x^n}\right)^p = \sqrt[m]{x^{n*p}}$$

- **RADICAL OF A RADICAL**
  The radicand stays the same, and the index is the product of the indices.

  $$\sqrt[m]{\sqrt[n]{x}} = \sqrt[m*n]{x}$$

Sample Problem 5: Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\left(\sqrt[5]{125}\right)^2 =$

b. $\sqrt[3]{\sqrt{x^5}} =$

Like radicals are radicals that have the same index and the same radicand

Sample Problem 6: Simplify radicals and recognize like or unlike radicals.

a. $2\sqrt{5}$ *and* $3\sqrt{5}$

b. $\sqrt{125}$; $3\sqrt{80}$ *and* $\sqrt{45}$

c. $4\sqrt[3]{x^2}$ *and* $4\sqrt{x^3}$

d. $3\sqrt{ab^2}$; $4b\sqrt{a}$; $\sqrt{4ab^2}$