Algebra1Coach.com

Simplifying Radicals

Unit 10 Lesson 2

Simplifying Radicals

Students will be able to:

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number
- Simplify expressions involving radicals using the properties of radicals.

Simplifying Radicals

Key Vocabulary:

- Radical
- Radicand
- Simplifying Radicals
- Denominator
- Like Radicals

Simplifying Radicals

Radicals (or roots) are the opposite operation of applying exponents.

A power can be undone with a radical and a radical can be undone with a power.

- $\mathbf{2^2 = 4 \quad so \ \sqrt{4} = 2}$
- $\mathbf{2^3 = 8 \quad so \ \sqrt[3]{8} = 2}$

$$\sqrt[index]{\mathbf{radicand}}$$

Radical sign

Radicals (or roots) are the opposite operation of applying exponents.
A power can be undone with a radical and a radical can be undone with a power.
$2^2 = 4 \quad so \ \sqrt{4} = 2$ $\sqrt[index]{radicand}$ Radical sign
$2^3 = 8 \quad so \ \sqrt[3]{8} = 2$

Simplifying Radicals

Common Radicals:

- A square (second) root is written as $\sqrt{\ }$
- A cube (third) root is written as $\sqrt[3]{\ }$
- A fourth root is written as $\sqrt[4]{\ }$

Simplifying Radicals

- To simplify a radical, factor the expression under the radical sign to its prime factors.
- For every pair of like factors, bring out one of the factors.
- Multiply whatever is outside the sign, and then multiply whatever is inside the sign.
- Remember that for each pair, you “bring out” only one of the numbers.

Simplifying Radicals

- Variables in a radicand are simplified in the same way. You've got a pair of can be taken "out front".
- Negative Radicals

The only restriction that exists for negative signs and radicals is that there cannot be a negative sign under an even root since there is no real solution to this problem.

However, a negative sign can exist in front of a radical or under odd roots and still be able to obtain a real number.

Simplifying Radicals

- **PRODUCT RULE FOR RADICAL**

The radical of the product is the product of two radicals.

$$\sqrt[m]{xy} = \sqrt[m]{x} * \sqrt[m]{y}$$

Simplifying Radicals

QUOTIENT RULE FOR RADICAL

The radical of a quotient is the quotient of the radicals.

$$\sqrt[m]{\frac{x}{y}} = \frac{\sqrt[m]{x}}{\sqrt[m]{y}} \qquad y \neq 0$$

Simplifying Radicals

Sample Problem 1: Simplify the following expressions. Assume that all variables represent positive real numbers.

$\boldsymbol{a}.\ \sqrt{\mathbf{20}} =$

$\boldsymbol{b}.\sqrt{\mathbf{72}\boldsymbol{a}^{\mathbf{2}} =}$

$\mathbf{c}.\sqrt{\mathbf{45} =}$

Simplifying Radicals

Sample Problem 1: Simplify the following expressions. Assume that all variables represent positive real numbers.

$\boldsymbol{a}.\ \sqrt{20} = \sqrt{4 * 5} = \sqrt{2^2} * \sqrt{5} = 2\sqrt{5}$

$\boldsymbol{b}.\ \sqrt{72a^2} = \sqrt{2 * 36 * a^2} = \sqrt{2} * \sqrt{6^2} * \sqrt{a^2} = 6a\sqrt{2}$

$\mathbf{c}.\ \sqrt{45} = \sqrt{3^2 * 5} = \sqrt{3^2} * \sqrt{5} = 3\sqrt{5}$

Simplifying Radicals

Sample Problem 1: Simplify the following radicals. Assume that all variables represent positive real numbers.

d. $\sqrt{\frac{24}{54}} =$

e. $\sqrt[5]{\frac{-32y^5}{x^{10}}} =$

f. $\sqrt[3]{\frac{xz^4}{y^6}} =$

Simplifying Radicals

Sample Problem 1:

Simplify the following radicals. Assume that all variables represent positive real numbers.

d. $\sqrt{\frac{24}{54}} = \frac{\sqrt{24}}{\sqrt{54}} = \frac{\sqrt{4*6}}{\sqrt{9*6}} = \frac{\sqrt{2^2}}{\sqrt{3^2}} = \pm\frac{2}{3}$

e. $\sqrt[5]{\frac{-32y^5}{x^{10}}} = \frac{\sqrt[5]{(-2)^5 * y^5}}{\sqrt[5]{x^5} * \sqrt[5]{x^5}} = \frac{-2y}{x^2}$

f. $\sqrt[3]{\frac{xz^4}{y^6}} = \frac{\sqrt[3]{xz^4}}{\sqrt[3]{y^6}} = \frac{\sqrt[3]{x} * \sqrt[3]{z} * \sqrt[3]{z^3}}{\sqrt[3]{y^3} * \sqrt[3]{y^3}} = \frac{z\sqrt[3]{xz}}{y^2}$

Simplifying Radicals

- Radicals can also be written in exponent notation. However, in this case the exponent would be a fraction.

$$\sqrt[m]{x^n} = x^{\frac{n}{m}}$$

- If n is odd then $\sqrt[n]{x^n} = x$
- If n is even then $\sqrt[n]{x^n} = |x|$

Simplifying Radicals

Sample Problem 2:

Write each expression in radical notation.

a. $(4)^{\frac{1}{2}} =$

b. $(-8)^{\frac{1}{3}} =$

c. $4^{\frac{1}{3}} =$

Simplifying Radicals

Sample Problem 2:

Write each expression in radical notation.

a. $(4)^{\frac{1}{2}} = \sqrt{4} = \pm 2$

b. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

c. $4^{\frac{1}{3}} = \sqrt[3]{4}$

Simplifying Radicals

Sample Problem 3:

Write each expression in exponential notation.

a. $\sqrt[3]{(-64)} =$

b. $\sqrt[5]{32} =$

c. $\sqrt[4]{625} =$

Simplifying Radicals

Sample Problem 2:

Write each expression in exponential notation.

a. $\sqrt[3]{(-64)} = (-64)^{\frac{1}{3}} = (-4)^{3*\frac{1}{3}} = -4$

b. $\sqrt[5]{32} = 2^{5*\frac{1}{5}} = 2$

c. $\sqrt[4]{625} = \sqrt[4]{5^4} = \pm 5^{\frac{4}{4}} = \pm 5$

Simplifying Radicals

RATIONALIZING THE DENOMINATOR

When dealing with fractions, a final answer cannot contain radicals in the denominator.

Therefore, it is necessary to eliminate any radical from the denominator.

The process of removing the radical from the denominator is called **rationalizing the denominator**.

Simplifying Radicals

Rationalizing the denominator

1. An expression is considered simplified only if there is no radical sign in the denominator.

$$\frac{1}{\sqrt[n]{x^m}} = \frac{1}{\sqrt[n]{x^m}} * \frac{\sqrt[n]{x^{n-m}}}{\sqrt[n]{x^{n-m}}} = \frac{\sqrt[n]{x^{n-m}}}{x}$$

2. Rationalizing the denominator with two terms, one or both of which involve square root.

Step 1: Multiply the numerator and denominator by the conjugate of the denominator

Step 2: Simplify the resulting expression if possible

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\frac{7}{\sqrt{5}} =$

b. $\frac{2}{\sqrt{(x+y)}} =$

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\frac{7}{\sqrt{5}} = \frac{7}{\sqrt{5}} * \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{5}$

b. $\frac{2}{\sqrt{(x+y)}} = \frac{2}{\sqrt{(x+y)}} * \frac{\sqrt{(x+y)}}{\sqrt{(x+y)}} = \frac{2\sqrt{(x+y)}}{(x+y)}$

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

c. $\frac{2}{2-\sqrt{6}} =$

d. $\frac{x}{1-\sqrt{x}} =$

Simplifying Radicals

Sample Problem 4:

Simplify the following expressions. Assume that all variables represent positive real numbers.

c. $\frac{2}{2-\sqrt{6}} = \frac{2}{2-\sqrt{6}} * \frac{2+\sqrt{6}}{2+\sqrt{6}} = \frac{2(2+\sqrt{6})}{2^2-(\sqrt{6})^2} = \frac{2(2+\sqrt{6})}{-2} = -(2+\sqrt{6})$

d. $\frac{x}{1-\sqrt{x}} = \frac{x}{1-\sqrt{x}} * \frac{1+\sqrt{x}}{1+\sqrt{x}} = \frac{x(1+\sqrt{x})}{1^2-(\sqrt{x})^2} = \frac{x(1+\sqrt{x})}{1-x}$

Simplifying Radicals

- **POWER OF A RADICAL**

The radicand has to be raised to the exponent of the power while the index remains the same.

$$\left(\sqrt[m]{x^{n}}\right)^{p} = \sqrt[m]{x^{n*p}}$$

Simplifying Radicals

- **RADICAL OF A RADICAL**

The radicand stays the same, and the index is the product of the indices.

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[m*n]{x}$$

Simplifying Radicals

Sample Problem 5: Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\left(\sqrt[5]{125}\right)^2 =$

b. $\sqrt[3]{\sqrt{x^5}} =$

Simplifying Radicals

Sample Problem 5: Simplify the following expressions. Assume that all variables represent positive real numbers.

a. $\left(\sqrt[5]{125}\right)^2 = \sqrt[5]{5^{3*2}} = \sqrt[5]{5^6} = 5\sqrt[5]{5}$

b. $\sqrt[3]{\sqrt{x^5}} = \sqrt[3*2]{x^5} = \sqrt[6]{x^5}$

Simplifying Radicals

Like radicals are radicals that have the same index and the same radicand

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

a. $\mathbf{2\sqrt{5}}$ **and** $\mathbf{3\sqrt{5}}$

b. $\mathbf{\sqrt{125}}$; $\mathbf{3\sqrt{80}}$ **and** $\mathbf{\sqrt{45}}$

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

a. $2\sqrt{5}$ and $3\sqrt{5}$

$LIKE\ RADICALS$

b. $\sqrt{125}$; $3\sqrt{80}$ and $\sqrt{45}$

$\sqrt{25 * 5}$; $3\sqrt{16 * 5}$; and $\sqrt{9 * 5}$

$5\sqrt{5}$; $12\sqrt{5}$; and $3\sqrt{5}$

$LIKE\ RADICALS$

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

c. $4\sqrt[3]{x^2}$ $\boldsymbol{and}$ $4\sqrt{x^3}$

d. $3\sqrt{ab^2}$; $4b\sqrt{a}$; $\sqrt{4ab^2}$

Simplifying Radicals

Sample Problem 6:

Simplify radicals and recognize like or unlike radicals.

c. $\boldsymbol{4\sqrt[3]{x^2}\ \ and\ \ 4\sqrt{x^3}}$

$\boldsymbol{UNLIKE\ RADICALS}$

d. $\boldsymbol{3\sqrt{ab^2};\quad 4b\sqrt{a};\quad \sqrt{4ab^2}}$

$\boldsymbol{3b\sqrt{a}\ ;\quad 4b\sqrt{a};\quad 2b\sqrt{a}}$

$\boldsymbol{LIKE\ RADICALS}$