**THE SET OF REAL NUMBERS**

When we were young, we were taught how to count using the set of counting numbers.

**{1, 2, 3, 4, 5, 6, …}**

Little did we know that numbers too have different types. The tree diagram below shows the different types numbers and how each kind is related to one another.

**REAL NUMBERS**

Apparently, any number that you can think of are called **REAL NUMBERS**. These are the set of numbers that is formed by combining the **rational numbers** and the **irrational numbers**.

**Examples:**

$$π, e, \frac{22}{7}, \sqrt{2}, \sqrt{3}, \sqrt{7},\frac{3}{4},\frac{27}{11}, 9i, -2, -1, 0, 1, 2, 3$$

**REAL NUMBERS can be IRRATIONAL or RATIONAL.**

**IRRATIONAL NUMBERS**

**Irrational** means “**not rational**”. These are the set of all numbers whose decimal representation are neither terminating nor repeating. It cannot be expressed as a quotient of integers. These numbers cannot be expressed as a ratio of two numbers

**Examples:**

$$π, e, \frac{22}{7}, \sqrt{2}, \sqrt{3}, \sqrt{7 } $$

**RATIONAL NUMBERS**

These are the set of all numbers which can be expressed in the form: $\frac{a}{b}$, where $a$ and $b$ are integers and $b$ is not equal to $0$, written as $b\ne 0$. It can be expressed as **terminating** or **repeating** decimals.

**Examples:**

$$\frac{3}{4}, \frac{27}{11}, -2, -1, 0, 100, -25, 3.75$$

**RATIONAL NUMBERS can be NON-INTEGERS or INTEGERS.**

**NON-INTEGERS**

These are the set of all numbers that is neither a positive whole number, nor a negative whole number, nor zero. These include **decimals**, **fractions**, and **imaginary numbers**.

**Examples:**

$$\frac{3}{4},\frac{27}{11}, 9i, -\frac{1}{2}, -0.25, 1.75, \frac{5}{7}, 3\frac{2}{3} $$

**INTEGERS**

These are the set of numbers formed by **positive whole numbers**, **negative whole numbers**, and **zero**.

**Examples:**

$$…,-3,-2, -1, 0, 1, 2, 3,…$$

**INTEGERS can be NEGATIVE or WHOLE NUMBERS.**

**NEGATIVE INTEGERS**

These are whole numbers **less than zero** and usually mean a value that is a deficit or shortage.

**Examples:**

$$…, -5, -4, -3,-2, -1$$

**WHOLE NUMBERS**

These are the set of numbers formed by adding **0** to the set of **natural numbers** (also called as counting numbers).

**Examples:**

$$0, 1, 2, 3, 4, 5, 6,7, 8,9, 10, 11,…$$

**WHOLE NUMBERS include ZERO and POSITIVE INTEGERS.**

**ZERO**

**Zero** denotes the absence of all magnitude or quantity.

$$0$$

**POSITIVE INTEGERS**

These are the set of numbers that include all **natural numbers** (also known as **counting numbers**)

**Examples:**

$$1, 2, 3, 4, 5, 6, 7, 8,…$$

**Sample Problem 1**: Look at the numbers inside the box and classify each according to the type of number described.

$-0.2          1          0.\overbar{4}          0.71771777177771…$$ $

$$π          3          7          41          56 -5 \frac{7}{8}, 0.454545…$$

$$0, -\frac{1}{2}, -100, 0.75, \sqrt{2}$$

|  |  |  |
| --- | --- | --- |
| **a.** | **Real Numbers** | $$-0.2, 1, 0.4, 0.71771777177771…, π, 3, 7, 41, 56, -5, \frac{7}{8},$$$$ 0.454545…, 0, -\frac{1}{2}, -100, 0.75, \sqrt{2} $$ |
| **b.** | **Irrational Numbers** | $$0.71771777177771…, π, \sqrt{2}$$ |
| **c.** | **Rational Numbers** | $$-0.2, 1, 0.4, 3, 7, 41, 56, -5, \frac{7}{8}, 0.454545…, 0, -\frac{1}{2}, -100, 0.75$$ |
| **d.** | **Non-Integers** | $$-0.2, 0.4, -5, \frac{7}{8}, 0.454545…,-\frac{1}{2}, 0.75$$ |
| **e.** | **Integers** | $$1, 3, 7, 41, 56, -5, 0, -100$$ |
| **f.** | **Negative Integers** | $$-5, -100$$ |
| **g.** | **Whole Numbers** | $$1, 3, 7, 41, 56, 0 $$ |
| **h.** | **Positive Integers** | $$1, 3, 7, 41, 56$$ |

**REAL NUMBERS ON THE NUMBER LINE**

A **NUMBER LINE** is a straight line with numbers written in equal intervals. It can be used to show the sets of **real numbers** composed of **rational** and **irrational numbers**. On a **REAL NUMBER LINE**:

* There is a point that corresponds for every real number.
* There is a real number for each point.

**0.5**

$$π$$

$$-\sqrt{2}$$

**-2**$\frac{2}{3}$

**-4.75**

**OPPOSITES**

The idea of opposites used in real-life can include, but are not limited to the following:

**Direction**

North or South

**Length**

Long Short or South

**Size**

Big or Small

**Temperature**

Warm or Cold

**Height**

Tall or Short

**Altitude**

Low or High

**Quantity**

Many or Few

**Color**

Bright or Dark

In Mathematics, on the other hand, OPPPOSITES are denoted by the following signs:

**Negative Sign**

This symbol is written before a number that is negative.

**Example:** $-7$ is read as “**negative 7**”

It is very important to write that symbol before a negative number to indicate that it is negative.

**Example:** **-10** is read as “**negative 10**”

$$-$$

$$+$$

**Positive Sign**

This symbol is written before a number that is positive.

**Example:** $+7$ is read as “**positive 7**”

If there no sign before a number, then that number is considered positive.

**Example:** **7** is understood to be “**positive 7**”

**Also, ZERO IS NEITHER POSITIVE NOR NEGATIVE.**

**REPRESENTATIONS OF OPPOSITES IN REAL LIFE**

|  |  |
| --- | --- |
| **POSITIVE** | **NEGATIVE** |
| An increase of $1 is denoted by $+1$. | A decrease of $1 is denoted by -$1$. |
| Walking 10 steps north is denoted by $+10.$ | Walking 10 steps south is denoted by $10.$ |
| An increase of 6 degrees in temperature is denoted by $+6$. | A decrease of 6 degrees in temperature is denoted by -$6$. |
| 5 feet above sea level is denoted by $5$. | 5 feet below sea level is denoted by $-5$. |
| A deposit of $5000 denotes $+5000$. | A withdrawal of $5000 denotes -$5000$. |

**Sample Problem 2**: Represent the following with integers.

|  |  |  |
| --- | --- | --- |
| a. | A weight loss of 7 kilograms | **-7** |
| b. | Walking 10 blocks north | **+10** |
| c. | 225 meters below sea level. | **-225** |
| d. | Going up the stairs by 6 steps | **+6** |
| e. | The temperature drops 5 degrees | **-5** |
| f. | Losing 10 points in a game | **-10** |
| g. | Moving a table 5 meters forward | **5** |
| h. | A debt of $10,000 | **-10,000** |

**INTEGERS ON THE NUMBER LINE**

Integers, composed of negative whole numbers, positive whole numbers and zero, can be graphed or plotted on a number line.

The starting point of a number line is at its origin, at **ZERO**.

**0**

**1**

**2**

**3**

**4**

**5**

**6**

**7**

**8**

**9**

**-1**

**-2**

**-3**

**-4**

**-5**

**-6**

**-7**

**-8**

**-9**

**Negative Intgers**

**Positive Integers**

**POSITIVE INTEGERS** on the number line are the integers that are found to the right of zero. As the number line extends to the right of zero, the integers increase.

**NEGATIVE INTEGERS** on the number line are the integers that are found to the left of zero. As the number line extends to the left of zero, the integers decrease.

$+4$ **is four units away to the right of 0.**

$-4$ **is four units away to the left of 0.**

**INCREASING**

**DECREASING**

**Sample Problem 3**: Graph the real numbers $-1, 3, 0,  2, \frac{3}{4}, -\frac{1}{2}$ and $-2.6$ on the number line and write the numbers in increasing order.

**Solution:**

**0**

**1**

**2**

**3**

**4**

**-1**

**-2**

**-3**

**-4**

$$-1$$

$$0$$

$$3$$

$$2$$

$$\frac{3}{4}$$

$$-\frac{1}{2}$$

$$-2.6$$

$$-2.6,  -1,  -\frac{1}{2},  0,  \frac{3}{4},  2,  3$$

**Sample Problem 4**: Plot the integers $-4$ and $-6$ on the number line and write two inequalities, using the symbols > or <, that compare the two numbers.

**Solution:**

**-4**

**-3**

**-2**

**-1**

**0**

**-5**

**-6**

**-7**

**-8**

$$-6$$

$$-4$$

$-6<-4$$-4>-6$

**Sample Problem 5**: Arrange the real numbers below in descending order.

$$-0.25, \frac{3}{4}, -\frac{1}{2}, 9, 0, -7, \frac{2}{3}, -3, 3, 1$$

**Solution:** $9, 3, 1, \frac{3}{4}, \frac{2}{3}, 0, -0.25, -\frac{1}{2}, -3, -7 $

**ABSOLUTE VALUE OF A REAL NUMBER**

**ABSOLUTE VALUE** of a real number is the **distance between the origin and the point representing the real number**. The symbol$ \left|x\right| $represents the absolute value of a number$ x$.

**0**

**1**

**2**

**3**

**4**

**5**

**6**

**-1**

**-2**

**-3**

**-4**

**-5**

**-6**

**5 units**

**5 units**

|  |  |
| --- | --- |
| $$\left|-5\right|=5$$**The distance of -5 to the origin is 5 units.** | $$\left|5\right|=5$$**The distance of 5 to the origin is 5 units.** |

**Sample Problem 6**: Evaluate and graph the numbers $\left|2.5\right|$ and $\left|-\frac{1}{2}\right|$ on the number line.

$$2.5$$

$$-\frac{1}{2}$$

$\frac{1}{2} $**units**

**2.5 units**

**0**

**1**

**2**

**3**

**4**

**-1**

**-2**

**-3**

**-4**

$\left|-\frac{1}{2}\right|=\frac{1}{2}units$$\left|2.5\right|=2.35units$

**Sample Problem 7**: Determine the value of each.

|  |  |  |
| --- | --- | --- |
| a. | $$\left|0.25\right|$$ | 0.25 |
| b. | $$\left|-9\right|$$ | 9 |
| c. | $$\left|-\frac{6}{5}\right|$$ | $$\frac{6}{5}$$ |
| d. | $$\left|-11\right|$$ | 11 |
| e. | $$\left|32\right|$$ | 32 |