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The Real Number System

Unit 1 Lesson 1

Students will be able to:

- Classify the set of real numbers using their properties and characteristics.
- Relate the concept of "opposites" in real-life situations.
- Represent the real-life situations with integers.
- Graph/Plot real numbers on a number line.
- Differentiate positive integers form negative integers.
- Graph/Plot integers on a number line.
- Compare real numbers on a number line.
- Arrange real numbers given a specific order.
- Define absolute value.
- Find the absolute value of real numbers.

Key Vocabulary:

Real Number
Irrational Number
Rational Number
Non-Integer
Integer
Whole Number
Counting / Natural Number
Positive Integers
Negative Integers
Number Line
Opposites
Absolute Value

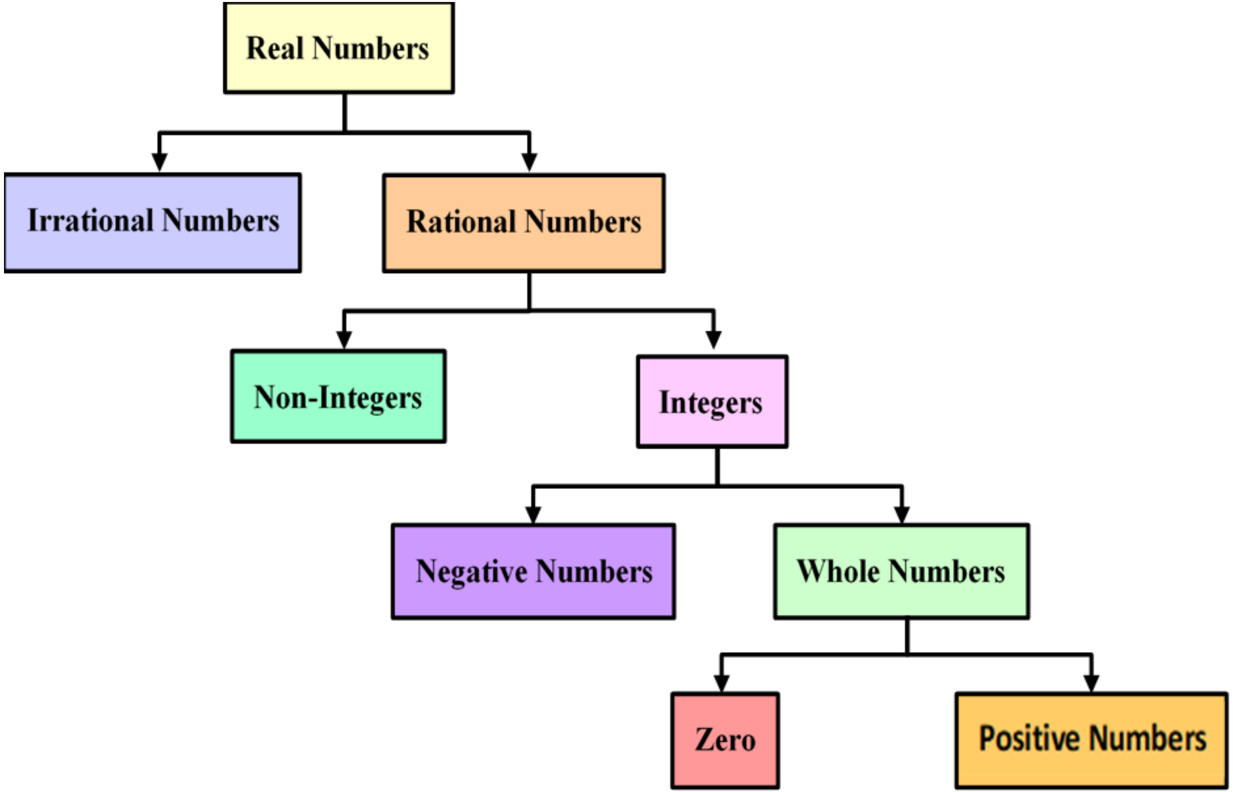
THE SET OF REAL NUMBERS

When we were young, we were taught how to count using the set of counting numbers.

$$\{1, 2, 3, 4, 5, 6, \ldots\}$$

Little did we know that numbers too have different types. The tree diagram below shows the different types numbers and how each kind is related to one another.

THE SET OF REAL NUMBERS



THE SET OF REAL NUMBERS

REAL NUMBERS

Apparently, any number that you can think of are called **REAL NUMBERS**. These are the set of numbers that is formed by combining the **rational numbers** and the **irrational numbers**.

Examples:

$$\pi,\ e,\ \frac{22}{7},\ \sqrt{2},\ \sqrt{3},\ \sqrt{7},\ \frac{3}{4},\ \frac{27}{11},\ 9i,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3$$

REAL NUMBERS can be IRRATIONAL or RATIONAL.

IRRATIONAL NUMBERS

Irrational means "not rational". These are the set of all numbers whose decimal representation are neither terminating nor repeating. It cannot be expressed as a quotient of integers. These numbers cannot be expressed as a ratio of two numbers

Examples:

$$\pi,\ e,\ \frac{22}{7},\ \sqrt{2},\ \sqrt{3},\ \sqrt{7}$$

REAL NUMBERS can be IRRATIONAL or RATIONAL.

RATIONAL NUMBERS

These are the set of all numbers which can be expressed in the form: $\frac{a}{b}$, where a and b are integers and b is not equal to 0, written as $b \neq 0$. It can be expressed as **terminating** or **repeating** decimals.

Examples:

$$\frac{3}{4}, \frac{27}{11}, -2, -1, 0, 100, -25, 3.75$$

RATIONAL NUMBERS can be NON-INTEGERS or INTEGERS.

NON-INTEGERS

These are the set of all numbers that is neither a positive whole number, nor a negative whole number, nor zero. These include **decimals**, **fractions**, and **imaginary numbers**.

Examples:

$$\frac{3}{4}, \frac{27}{11}, 9i, -\frac{1}{2}, -0.25, 1.75, \frac{5}{7}, 3\frac{2}{3}$$

RATIONAL NUMBERS can be NON-INTEGERS or INTEGERS.

INTEGERS

These are the set of numbers formed by **positive whole numbers, negative whole numbers**, and **zero.**

Examples:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

INTEGERS can be NEGATIVE or WHOLE NUMBERS.

NEGATIVE INTEGERS

These are whole numbers **less than zero** and usually mean a value that is a deficit or shortage.

Examples:

$$\ldots, -5, -4, -3, -2, -1$$

INTEGERS can be NEGATIVE or WHOLE NUMBERS.

WHOLE NUMBERS

These are the set of numbers formed by adding 0 to the set of natural numbers (also called as counting numbers).

Examples:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...

WHOLE NUMBERS include ZERO and POSITIVE INTEGERS.

ZERO

Zero denotes the absence of all magnitude or quantity.

0

WHOLE NUMBERS include ZERO and POSITIVE INTEGERS.

POSITIVE INTEGERS

These are the set of numbers that include all **natural numbers** (also known as **counting numbers**)

Examples:

1, 2, 3, 4, 5, 6, 7, 8, ...

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177717777 1\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

a. Real Numbers

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.71771777177771\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

b. Irrational Numbers

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

-0.2	1	$0.\overline{4}$	$0.71771777177771\ldots$				
π	3	7	41	56	-5	$\frac{7}{8},$	$0.454545\ldots$
$0,$	$-\frac{1}{2},$	$-100,$	$0.75,$	$\sqrt{2}$			

c. Rational Numbers

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.71771777177771\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

d. Non-Integers

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177717777 1\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

e. Integers

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.717717771777771\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

f. **Negative Integers**

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

	-0.2	1	$0.\overline{4}$	$0.71771777177771\ldots$				
π	3	7	41	56	-5	$\frac{7}{8},$	$0.454545\ldots$	
	$0,$	$-\frac{1}{2},$	$-100,$	$0.75,$	$\sqrt{2}$			

g. Whole Numbers

Sample Problem 1: Look at the numbers inside the box and classify each according to the type of number described.

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.71771777177771\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

h. Positive Integers

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177717777 1\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

a. **Real Numbers** $-0.2, 1, 0.4, 0.71771777177771\ldots, \pi, 3, 7, 41, 56, -5, \frac{7}{8},$ $0.454545\ldots, 0, -\frac{1}{2}, -100, 0.75, \sqrt{2}$

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177717777 1\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

b. Irrational Numbers $0.71771777177771\ldots, \pi, \sqrt{2}$

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177717777 1\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

c. **Rational Numbers** $-0.2, 1, 0.4, 3, 7, 41, 56, -5, \frac{7}{8}, 0.454545\ldots, 0, -\frac{1}{2}, -100, 0.75$

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177717777 1\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

d. Non-Integers $-0.2, 0.4, -5, \frac{7}{8}, 0.454545\ldots, -\frac{1}{2}, 0.75$

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177177771\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

e. Integers $1, 3, 7, 41, 56, -5, 0, -100$

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.71771777177771\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

f. **Negative Integers** $-5, -100$

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.7177177717777 1\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

g. Whole Numbers $1, 3, 7, 41, 56, 0$

Sample Problem 1: Solution

$-0.2 \quad 1 \quad 0.\overline{4} \quad 0.71771777177771\ldots$

$\pi \quad 3 \quad 7 \quad 41 \quad 56 \quad -5 \quad \frac{7}{8}, \quad 0.454545\ldots$

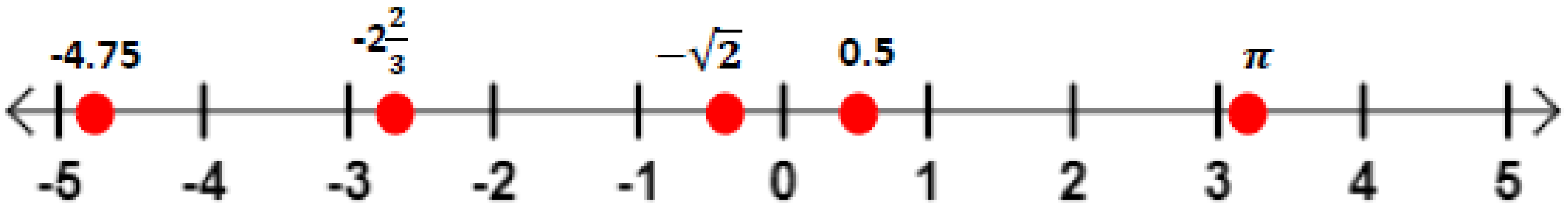
$0, \quad -\frac{1}{2}, \quad -100, \quad 0.75, \quad \sqrt{2}$

h. Positive Integers $1, 3, 7, 41, 56$

REAL NUMBERS ON THE NUMBER LINE

A **NUMBER LINE** is a straight line with numbers written in equal intervals. It can be used to show the sets of **real numbers** composed of **rational** and **irrational numbers**. On a **REAL NUMBER LINE**:

- There is a point that corresponds for every real number.
- There is a real number for each point.



OPPOSITES

The idea of opposites used in real-life can include, but are not limited to the following:

Direction
North or South

Length
Long Short or South

Size
Big or Small

Temperature
Warm or Cold

Height
Tall or Short

Altitude
Low or High

Quantity
Many or Few

Color
Bright or Dark

OPPOSITES

In Mathematics, on the other hand, OPPPOSITES are denoted by the following signs:

Positive Sign

This symbol is written before a number that is positive.

Example: +7 is read as "**positive 7**"

If there no sign before a number, then that number is considered positive.

Example: 7 is understood to be "**positive 7**"

Negative Sign —

This symbol is written before a number that is negative.

Example: −7 is read as "**negative 7**"

It is very important to write that symbol before a negative number to indicate that it is negative.

Example: -10 is read as "**negative 10**"

OPPOSITES

Also, ZERO IS NEITHER POSITIVE NOR NEGATIVE.

REPRESENTATIONS OF OPPOSITES IN REAL LIFE

POSITIVE	NEGATIVE
An increase of \$1 is denoted by +1.	A decrease of \$1 is denoted by -1.
Walking 10 steps north is denoted by +10.	Walking 10 steps south is denoted by 10.
An increase of 6 degrees in temperature is denoted by +6.	A decrease of 6 degrees in temperature is denoted by -6.
5 feet above sea level is denoted by 5.	5 feet below sea level is denoted by – 5.
A deposit of \$5000 denotes +5000.	A withdrawal of \$5000 denotes -5000.

Sample Problem 2: Represent the following with integers.

a. A weight loss of 7 kilograms

b. Walking 10 blocks north

c. 225 meters below sea level.

d. Going up the stairs by 6 steps

Sample Problem 2: Represent the following with integers.

e. The temperature drops 5 degrees

f. Losing 10 points in a game

g. Moving a table 5 meters forward

h. A debt of $10,000

Sample Problem 2: Solution

a.	A weight loss of 7 kilograms	**-7**
b.	Walking 10 blocks north	**+10**
c.	225 meters below sea level.	**-225**
d.	Going up the stairs by 6 steps	**+6**

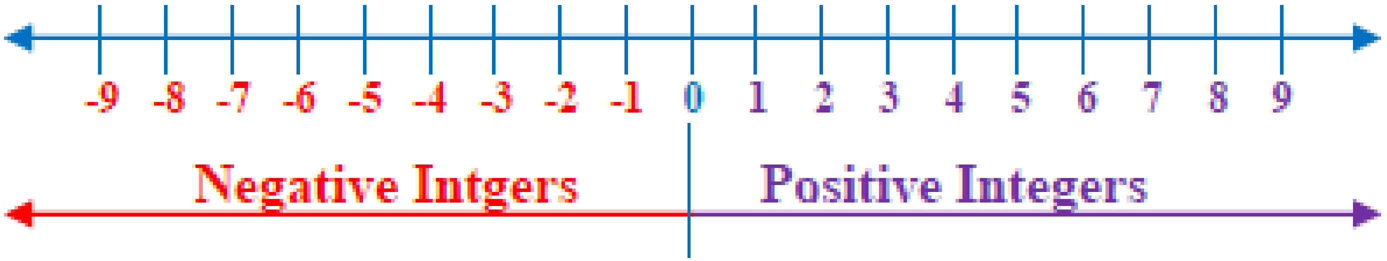
Sample Problem 2: Solution

e.	The temperature drops 5 degrees	**-5**
f.	Losing 10 points in a game	**-10**
g.	Moving a table 5 meters forward	**5**
h.	A debt of $10,000	**-10,000**

INTEGERS ON THE NUMBER LINE

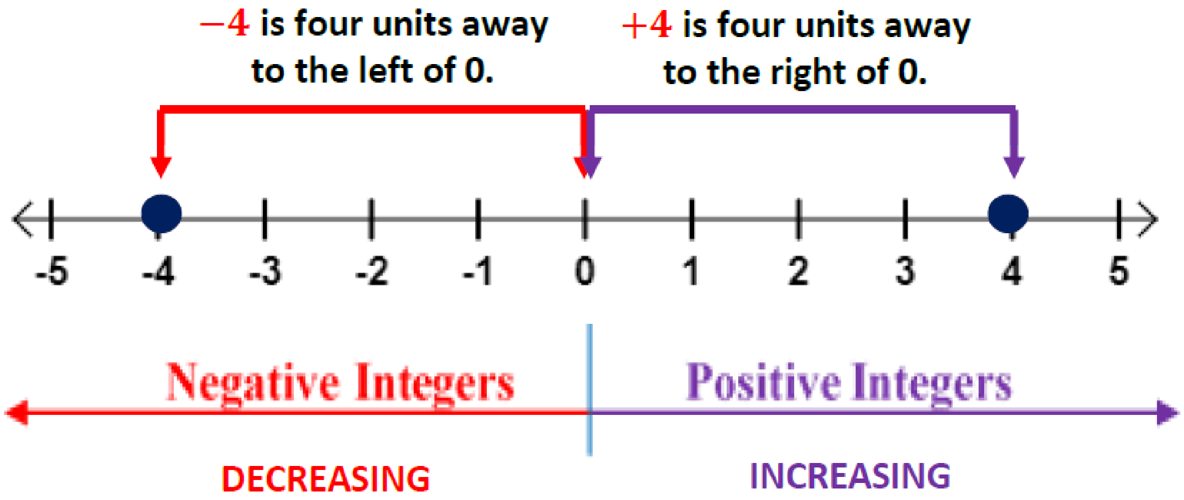
Integers, composed of negative whole numbers, positive whole numbers and zero, can be graphed or plotted on a number line.

The starting point of a number line is at its origin, at **ZERO**.



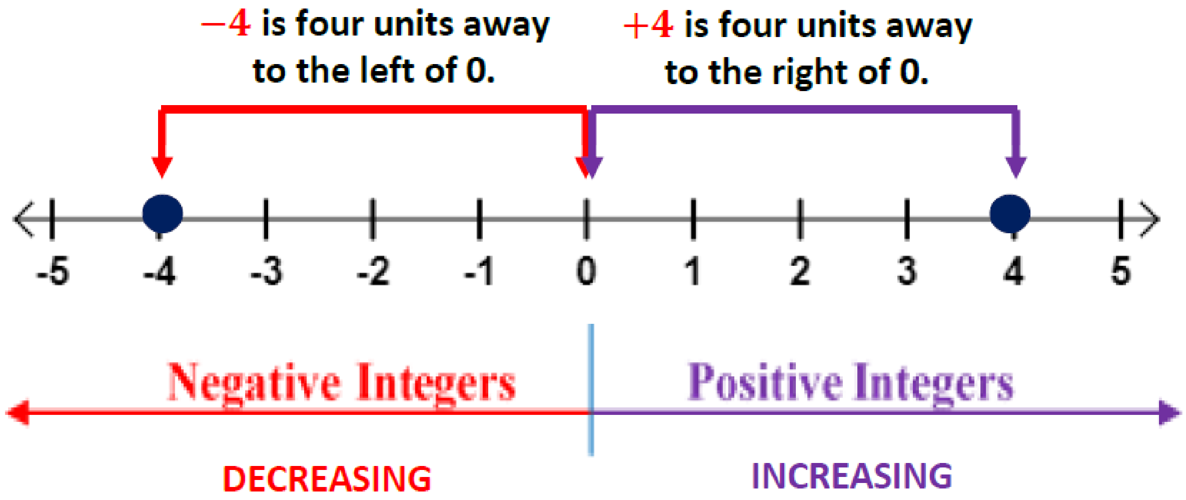
INTEGERS ON THE NUMBER LINE

POSITIVE INTEGERS on the number line are the integers that are found to the right of zero. As the number line extends to the right of zero, the integers increase.

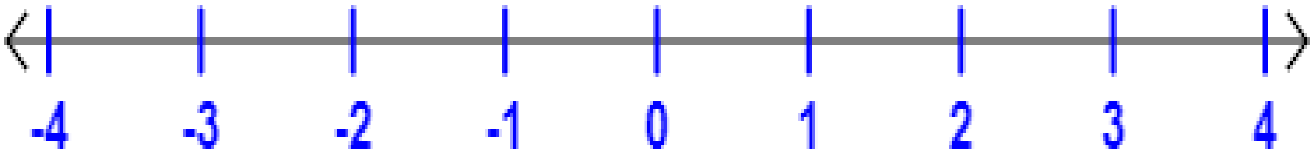


INTEGERS ON THE NUMBER LINE

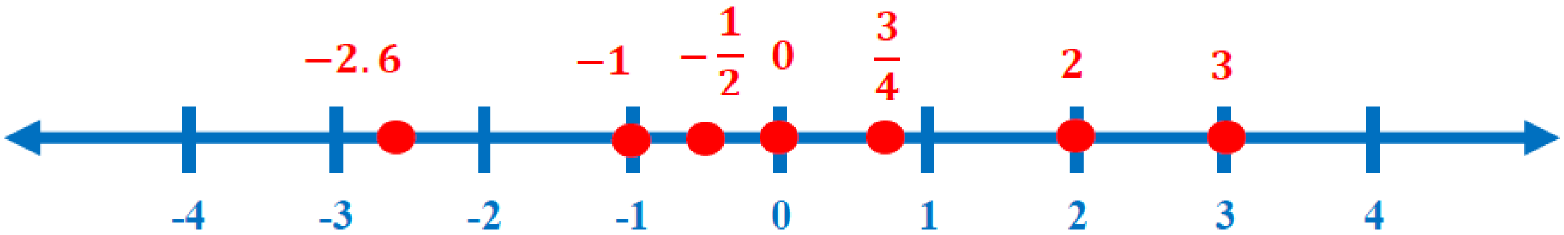
NEGATIVE INTEGERS on the number line are the integers that are found to the left of zero. As the number line extends to the left of zero, the integers decrease.



Sample Problem 3: Graph the real numbers $-1, 3, 0,\ 2, \frac{3}{4}, -\frac{1}{2}$ and -2.6 on the number line and write the numbers in increasing order.

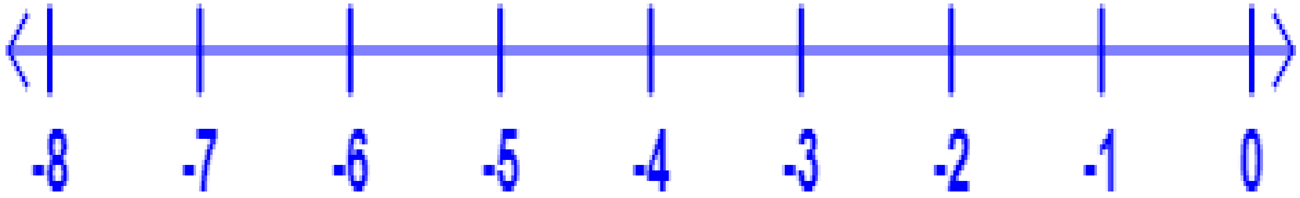


Sample Problem 3: Solution

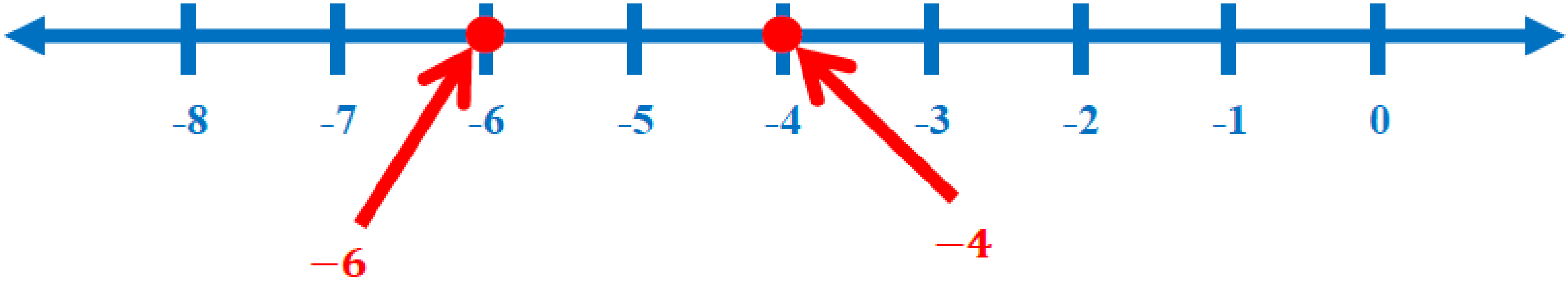


$$-2.6,\ -1,\ -\frac{1}{2},\ 0,\ \frac{3}{4},\ 2,\ 3$$

Sample Problem 4: Plot the integers -4 and -6 on the number line and write two inequalities, using the symbols > or <, that compare the two numbers.



Sample Problem 4: Solution



$-6 < -4$ $\qquad$ $-4 > -6$

Sample Problem 5: Arrange the real numbers below in descending order.

$$-0.25, \quad \frac{3}{4}, \quad -\frac{1}{2}, \quad 9, \quad 0, \quad -7, \quad \frac{2}{3}, \quad -3, \quad 3, \quad 1$$

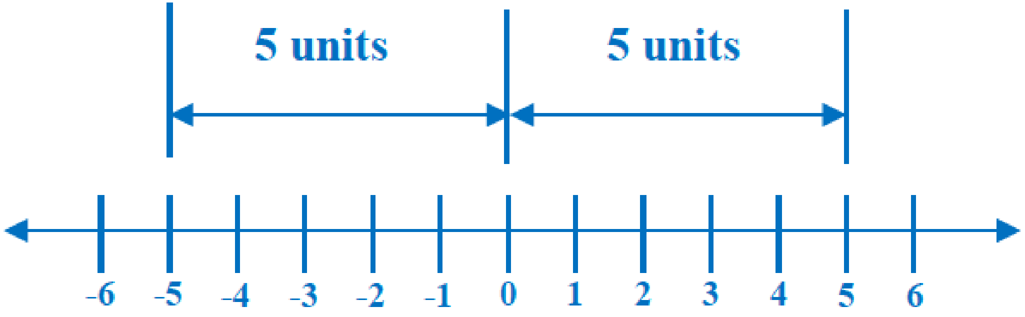
Sample Problem 5: Solution

$$-0.25, \quad \frac{3}{4}, \quad -\frac{1}{2}, \quad 9, \quad 0, \quad -7, \quad \frac{2}{3}, \quad -3, \quad 3, \quad 1$$

$$9,\ 3, 1, \frac{3}{4}, \frac{2}{3}, 0, -0.25, -\frac{1}{2}, -3, -7$$

ABSOLUTE VALUE OF A REAL NUMBER

ABSOLUTE VALUE of a real number is the **distance between the origin and the point representing the real number**. The symbol $|x|$ represents the absolute value of a number x.



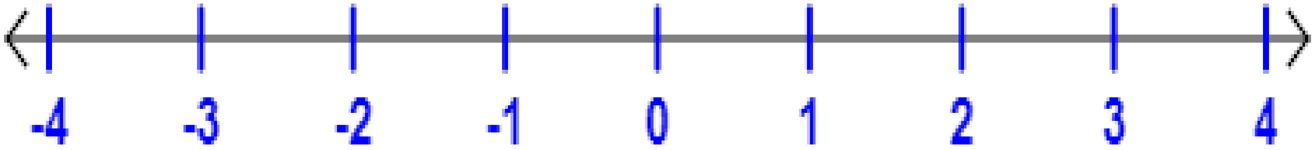
$|-5| = 5$

The distance of -5 to the origin is 5 units.

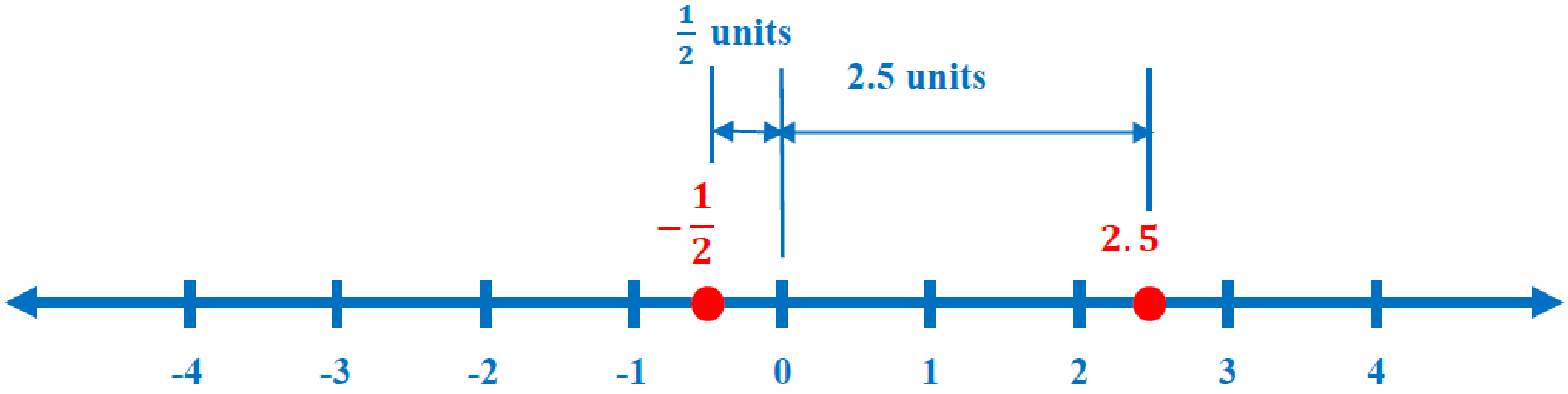
$|5| = 5$

The distance of 5 to the origin is 5 units.

Sample Problem 6: Evaluate and graph the numbers $|\mathbf{2.5}|$ and $\left|-\frac{\mathbf{1}}{\mathbf{2}}\right|$ on the number line.



Sample Problem 6: Solution



$\left|-\frac{1}{2}\right| = \frac{1}{2} units$ $|2.5| = 2.35 units$

Sample Problem 7: Determine the value of each.

a. $|0.25|$

b. $|-9|$

c. $\left|-\frac{6}{5}\right|$

d. $|-11|$

e. $|32|$

Sample Problem 7: Solution

a. $|0.25|$ $\quad$ 0.25

b. $|-9|$ $\quad$ 9

c. $\left|-\frac{6}{5}\right|$ $\quad$ $\frac{6}{5}$

d. $|-11|$ $\quad$ 11

e. $|32|$ $\quad$ 32