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Operations on Integers Guide Notes

OPERATIONS ON INTEGERS

Operations involving **positive** and **negative integers** (like the usual whole numbers that we know) also involve the four operations **ADDITION**, **SUBTRACTION**, **MULTIPLICATION** and **DIVISION**. Operations on integers can be done using the number line, or numerically using different rules.

ADDITION AND SUBTRACTION OF INTEGERS USING THE NUMBER LINE

A number line is used as a visual model to show what happens when positive and negative numbers are **added** or **subtracted**.

THINGS TO REMEMBER IN ADDING OR SUBTRACTING INTEGERS USING THE NUMBER LINE:

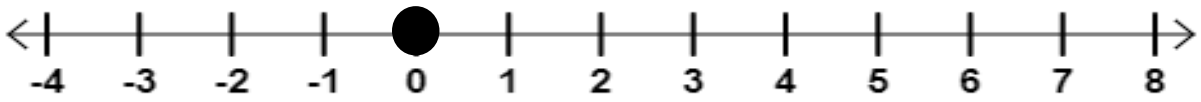
- The starting point is always at **ZERO**.
- If the integer is **positive** the movement on the number line is always to the **RIGHT**.
- If the integer is **negative** the movement on the number line is always to the **LEFT**.

ADDING SAME SIGNED NUMBERS

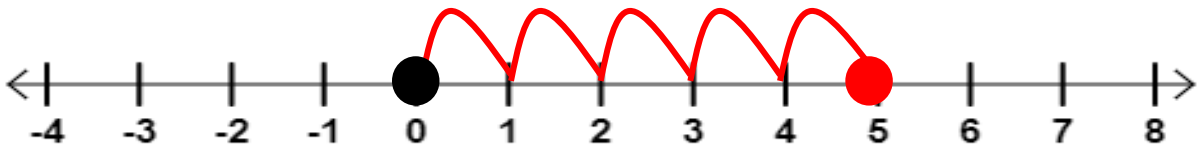
Example 1: $5 + 2$

Obviously, the answer is positive 7. Let's use a number line to visually show that the answer is really positive 7.

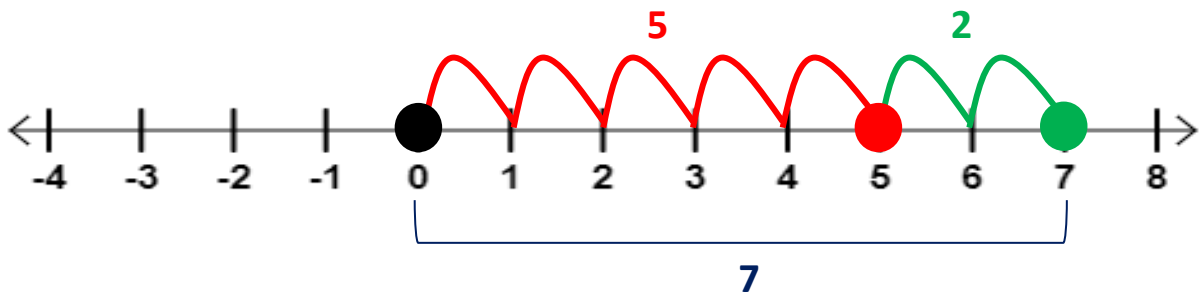
The starting point is at 0.



Since 5 is positive, move 5 units to the right of 0.



Since were adding positive 2 to positive 5, move 2 more units to the right of 5.



Therefore, $5 + 2 = 7$.

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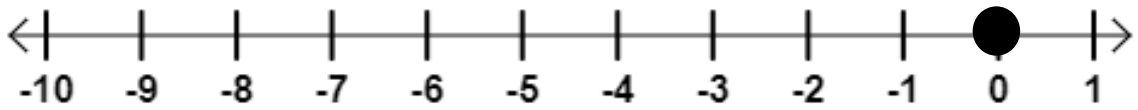
Operations on Integers Guide Notes

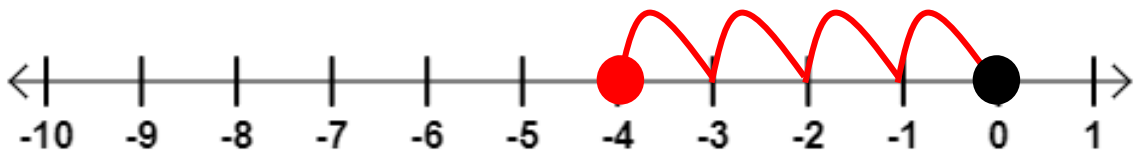
Example 2: $(-4) + (-3)$

Here, we are adding negative 4 and negative 3.

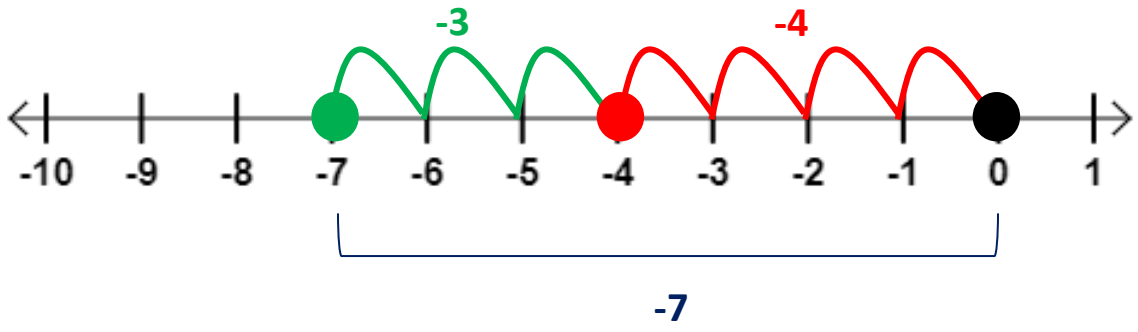
The starting point is at 0.



Since 4 is negative, move 4 units to the left of 0.



Since we are adding negative 3 to negative 4, move 3 units to the left of -4.



Therefore, $(-4) + (-3) = -7$.

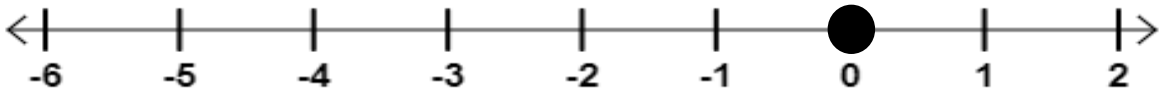
ADDING DIFFERENT SIGNED NUMBERS

Adding different signed numbers means that the addends have different signs. It's when you add a negative number to a positive number; or add a positive number to a negative number. The same rules apply if the number is positive, move to the right; and if the number is negative, move to the left.

Example 1: $(-5) + 2$

Here we are adding positive 2 to negative 5.

The starting point is at 0.



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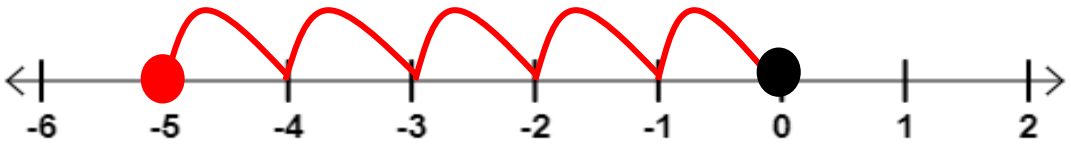
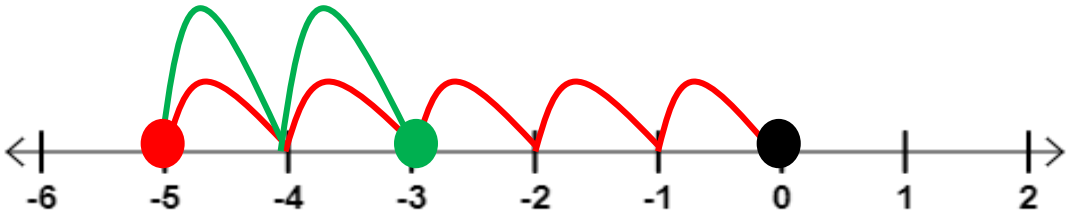
Operations on Integers Guide Notes

Since 5 is negative, move 5 units to the left of 0.



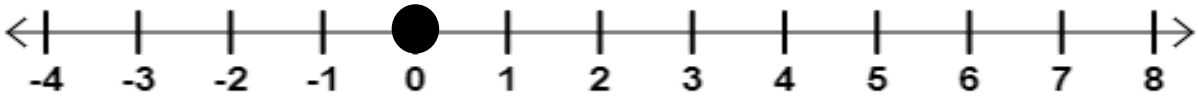
Since we are adding positive 2 to negative 5, we move 2 units to the right of -5.



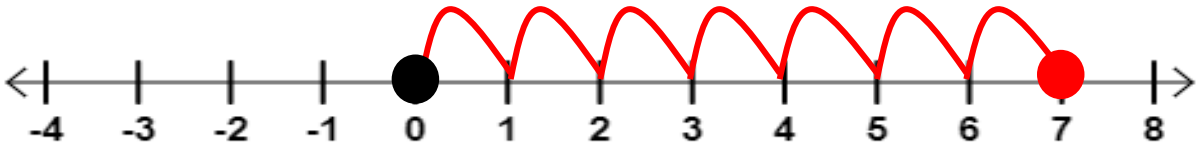
Example 2: $\mathbf{7 + (-3)}$

Here we are adding negative 3 to positive 7.

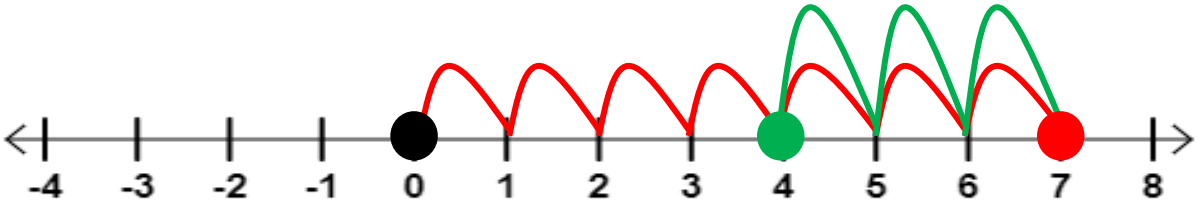
The starting point is at 0.



Since 7 is positive, move 7 units to the right of 0.



Since we are adding negative 3 to positive 7, we move 3 units to left of 7.



Therefore, $\mathbf{7 + (-3) = 4}$.

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Operations on Integers Guide Notes

SUBTRACTING INTEGERS

Before subtracting integers using the number line, there is one very important RULE to remember.

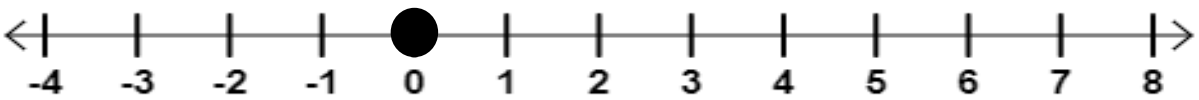
"TO SUBTRACT IS TO ADD ITS OPPOSITE"

$10 - 4 = ?$ Read as "positive 10 minus positive 4"	is the same as	$10 + (-4) = ?$ Read as "positive 10 plus negative 4"
$3 - 5 = ?$	is the same as	$3 + (-5) = ?$
$9 - 7 = ?$	is the same as	$9 + (-7) = ?$
$-3 - 2 = ?$ Read as "negative 3 minus positive 2"	is the same as	$-3 + (-2) = ?$ Read as "negative 3 plus negative 2"
$-4 - 6 = ?$	is the same as	$-4 + (-6) = ?$
$-1 - 7 = ?$	is the same as	$-1 + (-7) = ?$

Example 1: $3 - 5$

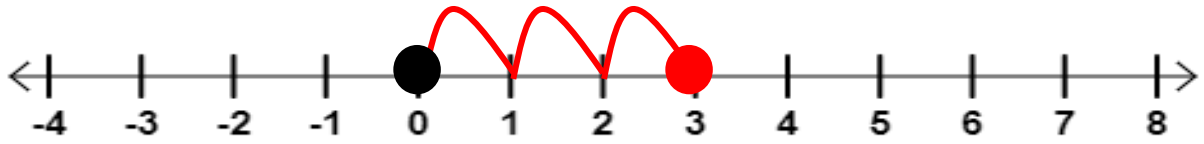
$3 - 5$ can be written as $3 + (-5)$.

The starting point is at 0.

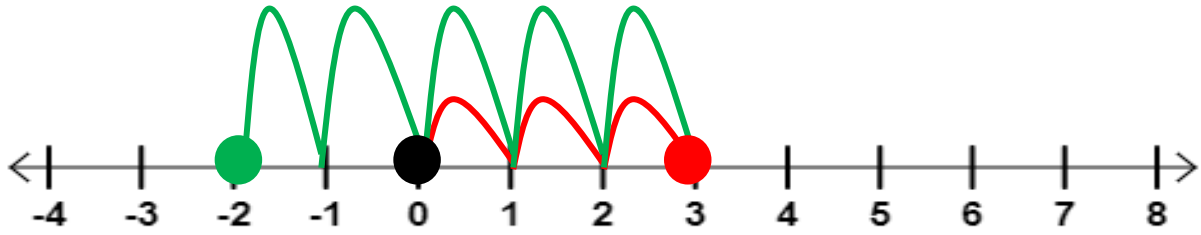


Operations on Integers Guide Notes

Since 3 is positive, move 3 units to the right of 0.



Since we are adding negative 5 to positive 3, move 5 units to the left of 3.

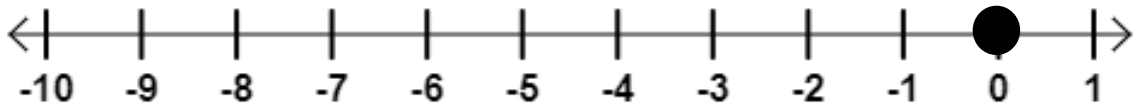


Therefore, $\mathbf{3 - 5 = -2}$

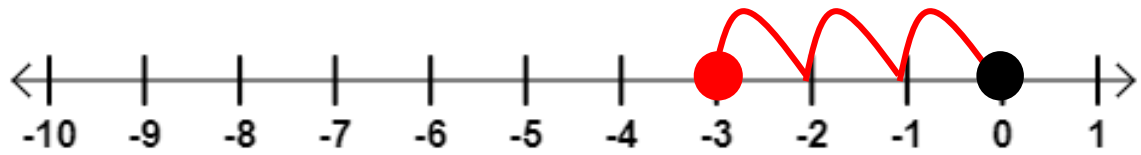
Example 2: $\mathbf{-3 - 2}$

$\mathbf{-3 - 2}$ can be written as $\mathbf{-3 + (-2)}$.

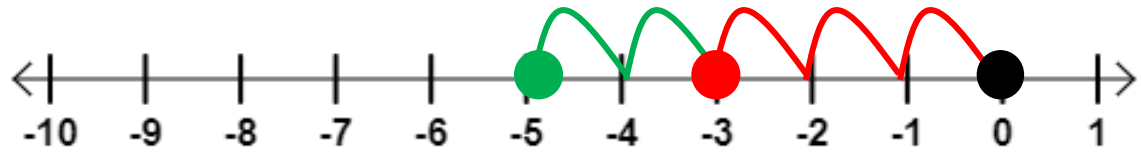
The starting point is at 0.



Since 3 is negative, move 3 units to the left of 0.



Since 2 is negative, move 2 units to the left of -3.



Therefore, $\mathbf{-3 - 2 = -5}$

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Operations on Integers Guide Notes

Sample Problem 1: Perform the indicated operation using the number line.

1. 1.$-10+6$

2. $-3+5$

3. $-7-5$

4. $6-10$

ADDING INTEGERS NUMERICALLY

Creating number lines to perform operations on integers can be a tedious task; but it is a nice way to visually show what happens to integers when they are added or subtracted. To perform operations easier and simpler, we can also do this numerically, using a few sets of rules.

ADDING SAME SIGNED INTEGERS

RULE: Simply add the given numbers and then copy the sign of the addends.

Getting the sum of integers having the same signs is like doing a simple addition. Just add the numbers and copy the sign of the addends.

Adding Positive Numbers				
Positive Number	+	Positive Number	=	Positive Number
3	+	6	=	9

Adding Negative Numbers				
Negative Number	+	Negative Number	=	Negative Number
(-7)	+	(-1)	=	-8

More Examples:

1. $\mathbf{5+19=24}$

2. $\mathbf{(-21)+(-19)=-40}$

3. $\mathbf{(-17)+(-31)+(-5)=-53}$

4. $\mathbf{24+42+8=74}$

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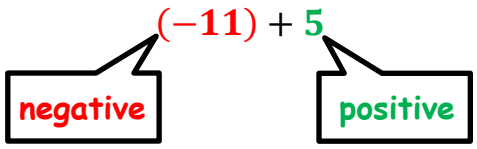
Operations on Integers Guide Notes

ADDING DIFFERENT SIGNED INTEGERS

This means that the addends have different signs.



To get the sum, follow this rule:

RULE: Subtract the absolute value of the smaller number from the absolute value of the larger number and then keep the sign of the number with the larger absolute value.

The **absolute value** of a number is the distance of the number from zero, without any regard to its direction. So it is safe to say that the absolute value of any integer is always positive (because there is **NO NEGATIVE DISTANCE**).

Example 1:

$$(-11) + 5 = ?$$

The absolute value of **-11** is **11**.

The absolute value of **5** is **5**

Subtracting the absolute value of smaller integer from the bigger integer.

$$11 - 5 = 6$$

Keep the sign of the of the integer with the larger absolute value. The integer with the larger absolute value is 11, and its sign is negative, thus the answer will carry the negative sign.

$$(-11) + 5 = -6$$

Or to make it even simpler, think of these integers as "**the money you have**" or "**the money you owe**".

Positive integers represent "the money you have".

Negative integers represent "the money you owe".

$$(-11) + 5 = -6$$

You owe somebody 11 and you have 5, therefore you still owe somebody 6.

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Operations on Integers Guide Notes

Example 2: Add $10 + (-2)$

The absolute value of **10** is **10**.

The absolute value of **-2** is **2**.

Subtracting the absolute value of smaller integer from the bigger integer.

$$10 - 2 = 8$$

Keep the sign of the of the integer with the larger absolute value. The integer with the larger absolute value is 10, and its sign is positive, thus the answer will carry the positive sign.

$$10 + (-2) = 8$$

To make you understand the problem better:

You have 10 and you owe somebody 2, therefore you still have 8.

More Examples:

1. $-5 + 20 = 15$

2. $10 + (-16) = -6$

3. $10 + (-5) + (-5) = 10 + (-5) + (-5)$
$= 5 + (-5)$
$= 0$

4. $(-16) + 11 + (-10) = (-16) + 11 + (-10)$
$= -5 + (-10)$
$= -15$

Sample Problem 2: Perform the indicated operation.

1. $16 + (-82)$

2. $(-17) + 25$

3. $-47 + 19$

4. $18 + (-55)$

5. $(-16) + 24 + (-8)$

6. $25 + (-35) + 5$

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Operations on Integers Guide Notes

SUBTRACTING INTEGERS NUMERICALLY

Just like adding integers numerically, subtracting integers also follow specific steps. These include the following:

Step 1: Copy the first number.

Step 2: Change subtraction into addition.

Step 3: Change the second number into its opposite.

Step 4: Apply the rules in adding integers.

SUBTRACTING SAME SIGNED INTEGERS

Example 1: $5 - 10$

The expression is read as "positive five minus positive ten".

Step 1: Copy the first number.

$$5$$

Step 2: Change subtraction into addition.

$$5 +$$

Step 3: Change the second number into its opposite.

$$5 + (-10)$$

Step 4: Apply the rules in adding integers.

$$5 + (-10) = -5$$

Example 2: $-8 - (-12)$

The expression is read as "negative eight minus negative twelve".

Step 1: Copy the first number.

$$-8$$

Step 2: Change subtraction into addition.

$$-8 +$$

Step 3: Change the second number into its opposite.

$$-8 + 12$$

Step 4: Apply the rules in adding integers.

$$-8 + 12 = 4$$

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Operations on Integers Guide Notes

SUBTRACTING DIFFERENT SIGNED INTEGERS

Example 1: $-11-20$

The expression is read as "negative eleven minus positive twenty".

Step 1: Copy the first number.

$$-11$$

Step 2: Change subtraction into addition.

$$-11+$$

Step 3: Change the second number into its opposite.

$$-11+(-20)$$

Step 4: Apply the rules in adding integers.

$$-11+(-20)=-31$$

Example 2: $7-(-17)$

The expression is read as "positive seven minus negative seventeen".

Step 1: Copy the first number.

$$7$$

Step 2: Change subtraction into addition.

$$7+$$

Step 3: Change the second number into its opposite.

$$7+17$$

Step 4: Apply the rules in adding integers.

$$7+17=24$$

More Examples:

1. $-19-25=-19+(-25)$
   $=-44$

2. $-11-(-9)=-11+9$
   $=-2$

3. $-32-6=-32+(-6)$
   $=-38$

4. $28-(-2)=28+2$
   $=30$

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Operations on Integers Guide Notes

Sample Problem 3: Perform the indicated operation.

1. $16 - 20$
2. $-11 - 29$
3. $3.14 - (-15)$
4. $-28 - (-6)$
5. $22 - (-38)$
6. $64 - 100$
7. $-72 - 22$
8. $64 - (-100)$

MULTIPLYING INTEGERS NUMERICALLY

The rules in multiplying integers is less complicated compared to adding and subtracting integers. Below are the rules.

Rule 1: The product of multiplying integers with the same signs is always **POSITIVE**.

Rule 2: The product of multiplying integers with different signs is always **NEGATIVE**.

MULTIPLYING SAME SIGNED INTEGERS

Example 1: $(\mathbf{5})(\mathbf{10})$

The expression is read as "**positive five times positive ten**".

Finding the product is the same as doing simple multiplication.

$$(\mathbf{5})(\mathbf{10}) = \mathbf{50}$$

Example 2: $(\mathbf{-16})(\mathbf{-4})$

The expression is read as "**negative sixteen times negative four**".

$$(\mathbf{-16})(\mathbf{-4}) = \mathbf{64}$$

MULTIPLYING DIFFERENT SIGNED INTEGERS

Example 1: $(\mathbf{-7})(\mathbf{4})$

The expression is read as "**negative 7 times positive 4**".

$$(\mathbf{-7})(\mathbf{4}) = \mathbf{-28}$$

Example 2: $(\mathbf{12})(\mathbf{-5})$

The expression is read as "**positive twelve times negative five**".

$$(\mathbf{12})(\mathbf{-5}) = \mathbf{-60}$$

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Operations on Integers Guide Notes

In multiplication involving more than two signed factors, multiply the integers one at a time to make sure that the answer is accurate. Of course, take note of the rules in multiplying integers.

Example: $(9)(-2)(+4)(5)$

$$(9)\ (-2)\ (-4)\ (-5)$$
$$(-18)\ (-4)\ (-5)$$
$$(72)(-5)$$
$$-360$$

More Examples:

1. $(9)(8) = 72$
2. $(-15)(-4) = 60$
3. $(-9)(11) = -99$
4. $(12)(-3) = -36$
5. $(4)(-2)(8) = (-8)(8) = -64$
6. $(5)(-6)(-2) = (-30)(-2) = 60$

Sample Problem 4: Perform the indicated operation.

1. $(7)(2)$
2. $(-6)(-12)$
3. $(-4)(-10)$
4. $(-21)(4)$
5. $(-1)(1)(-1)(-1)(2)(-1)$
6. $(-10)(2)(-3)(-1)$

DIVIDING INTEGERS NUMERICALLY

The rules in multiplying integers also work in dividing integers.

Rule 1: The quotient of dividing integers with the same signs is always **POSITIVE**.

Rule 2: The quotient of dividing integers with different signs is always **NEGATIVE**.

DIVIDING SAME SIGNED INTEGERS

Example 1: $16 \div 4$

The expression is read as "**positive sixteen divided by positive four**".

Finding the quotient is the same as doing simple division.

$$16 \div 4 = 4$$

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Operations on Integers Guide Notes

Example 2: $(-24) \div (-12)$

The expression is read as **"negative twenty-four divided by negative twelve".**

$$(-24) \div (-12) = 2$$

DIVIDING DIFFERENT SIGNED INTEGERS

Example 1: $(-8) \div 2$

The expression is read as **"negative eight divided by positive two".**

$$(-8) \div 2 = -4$$

Example 2: $(72) \div (-12)$

The expression is read as **"negative seventy-two divided by negative twelve".**

$$(72) \div (-12) = -6$$

More Examples:

1. $12 \div 4 = 3$
2. $-32 \div 8 = -4$
3. $-45 \div 15 = -3$
4. $88 \div (-22) = -4$

Sample Problem 5: Perform the indicated operation.

1. $-165 \div 15$
2. $-96 \div 8$
3. $-384 \div (-12)$
4. $-420 \div 15$

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